TECHNICAL INSTRUCTION
TT.5

Medium-Wave Impedance-Matching Networks.
Rejectors and Acceptors.

CONTENTS

INSTRUCTION TT.5
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MEDIUM-WAVE IMPEDANCE-MATCHING NETWORKS. REJECTORS AND ACCEPTORS.

SECTION A. INTRODUCTION.

This Instruction deals with the basic principles of design of medium-wave impedance-matching networks, with particular reference to circuits used in aerial transformer houses (A.T.H.) and with rejectors and acceptors.

The purpose of the matching network in an aerial transformer house is to bring about an impedance match between the aerial and the feeder which is used to convey the R.F. power from the transmitter to the aerial. As explained in Section C, page 7, the correct termination for the feeder is a non-reactive resistance, equal in value to the characteristic impedance of the feeder and usually very different from the impedance of the aerial.

In practice, the setting-up of such networks is done usually with the aid of measurements taken on an R.F. bridge, very exact calculations not being attempted owing to the influence of factors the magnitude of which is not readily predictable, e.g. stray capacities between components and from components to earth, stray mutual couplings, etc. Calculations of networks, even though approximate, are, however, most necessary in order to reduce the amount of trial and error work in setting-up and to ensure that the components used are properly rated.

Facility in handling problems involving network calculations is very largely a matter of practice. Certain worked examples are contained in the text and a number of problems in Appendix B. As the worked examples in the text are intended to illustrate basic principles the calculations are carried out to a higher degree of accuracy than would, in many instances, be necessary for practical purposes (or even warranted in some situations that occur in field work).

As a general rule it can be taken that the accuracy obtainable by reasonably careful use of a 10-in. slide rule is adequate.

The Operator j.

Frequent use of j is made in this Instruction and the following information is given for the benefit of those not familiar with its significance and application.

j will first be considered as a vector operator. The basic principles of vector representation of alternating quantities will be found in standard text books (e.g. Admiralty Handbook of Wireless Telegraphy), to which reference should be made, if necessary.

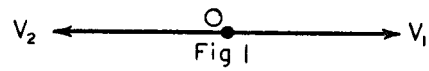


Fig 1

By the convention of vector diagrams OV_1 and OV_2 of Fig. 1 could represent two voltages, equal in value but 180° out of phase.

Also, by convention

$$OV_2 = -OV_1 = -1 \times OV_1 \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Thus -1 used in this manner can be regarded as an " operator " producing a phase shift of 180° in a vector quantity.

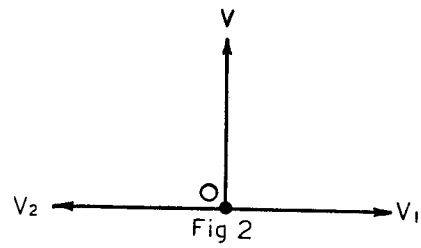


Fig 2

In Fig. 2 are shown the same two vectors OV_1 and OV_2, 180° out of phase with each other but, in addition, there is another vector OV, which represents a voltage of the same value as OV_1 but 90° out of phase with the latter.

What is the " operator " that would shift OV_1 90° to the position OV ?

Let j be the symbol for such an operator. Then,

$$jOV_1 = OV \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Since multiplication by j produces a shift of 90° in an anti-clockwise direction, it follows that :—

$$jOV = OV_2$$

But, from equation (2), $OV = jOV_1$

$$\therefore\ jOV = j(jOV_1) = j^2OV_1$$

$$\therefore\ OV_2 = j^2OV_1$$

From equation (1)

$$OV_2 = -1 \times OV_1.$$

Whence,

$$j^2 = -1$$

$$\therefore\ j = \sqrt{-1}$$

Thus, following on from vector conventions, j is

the 90° phase shift operator and, from the mathematical point of view, is equal to the square root of minus one.

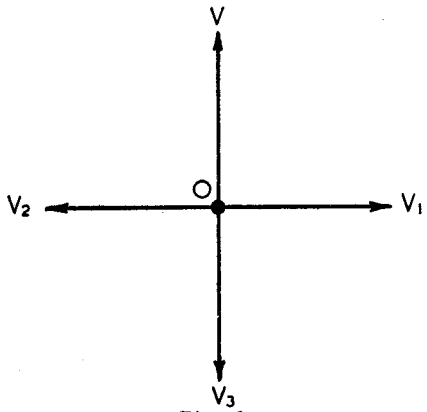


Fig 3

When considering 90° phase differences the matter of " lead " and " lag " arises and due regard must be paid to this in the application of operator j.

If in Fig. 3 $OV = jOV_1$, then $OV_3 = -jOV_1$.

This can be deduced as follows :—

$$OV_3 = jOV_2$$

But

$$OV_2 = -OV_1.$$

$$\therefore\ OV_3 = j(-OV_1) = -jOV_1.$$

Thus, if multiplication by +j produces a phase shift of 90° in an anti-clockwise sense, multiplication by −j will produce a phase shift of 90° in a clockwise sense.

j as an Algebraic Symbol.

The significance attached to j, as described above, does not in any way prevent the symbol being treated in equations and formulae in accordance with ordinary rules of algebra.

For example :—

$$(jA)\,(jB) = j^2AB$$
$$jA(-jB) = -j^2AB$$
$$\frac{jA}{jB} = \frac{A}{B}$$

j as $\sqrt{-1}$.

The square root of minus one is an imaginary quantity although the square of such a quantity is real.

From the fact that j is equal to $\sqrt{-1}$, it follows that :—

$$j^2 = -1$$
$$-j^2 = 1$$
$$(-j)^2 = -1$$
$$(-j)\,(+j) = 1$$
$$\frac{1}{j} = \frac{-j}{(j)\,(-j)} = -j$$
$$\frac{1}{-j} = \frac{j}{(-j)\,(j)} = +j$$
$$\frac{1}{j^2} = -1$$
$$\frac{1}{-j^2} = 1$$

The equalities given above are exceedingly useful for purposes of simplification of expressions involving j.

Complex Expressions.

A complex expression is one which contains both real and imaginary terms and is, in fact, of the general form A+jB.

A very useful fact to note is that if

$$A + jB = P + jQ$$

then

$$A = P$$

and

$$B = Q$$

Conjugate Expressions. Rationalisation.

A+jB and A−jB are said to be " conjugate " impedance expressions.

Note particularly the effect of multiplying together two conjugate expressions, thus :—

$$(A + jB)\,(A - jB) = A^2 - jAB + jAB + B^2$$
$$= A^2 + B^2$$

Advantage can be taken of the above to rationalise a complex denominator in a fractional expression.

Thus,

$$\frac{C}{A - jB} = \frac{C\,(A + jB)}{(A - jB)\,(A + jB)} = \frac{CA + jCB}{A^2 + B^2}$$
$$= \frac{CA}{A^2 + B^2} + j\,\frac{CB}{A^2 + B^2}$$

Application of j to Impedance Expressions.

In Fig. 4(a) is shown an impedance consisting of a resistance R, in series with an inductance L (assumed to have negligible resistance). V is the voltage across the combination of R and L, I the current flowing through R and L, V_R the voltage

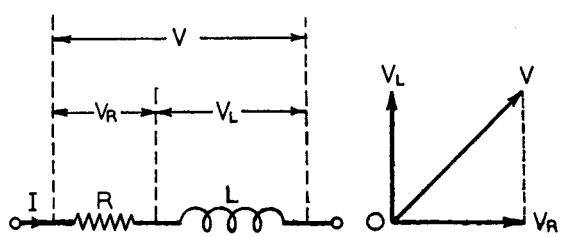


Fig 4

component across R and V_L the voltage component across L.

The voltages are represented in vector form in Fig. 4(b). From elementary considerations :—

$$V_R = RI \text{ volts.}$$

$$\text{and } V_L = \omega LI \text{ volts.}$$

V is not the arithmetical sum of V_R and V_L owing to the 90° phase difference between these two voltage components. V is, in fact, the " vectorial sum " of V_R and V_L (90° out of phase with V_R) which can be expressed in the following manner :—

$$V = V_R + jV_L$$

Calling the impedance of R and L in series Z, it follows that $ZI = RI + j\omega LI$.

Eliminating I gives :—

$$Z = R + j\omega L$$

In a similar manner it can be shown that if Z is made up of a resistance in series with a capacity,

$$Z = R - j\frac{1}{\omega C}$$

Impedance in General Terms.

In the text of this Instruction R, Z and X are used as general symbols for resistance, impedance and reactance respectively, appropriate suffix letters being added when it is required to give any particular signficance to the symbols, e.g. R_s and R_p, representing series and parallel connected resistances respectively.

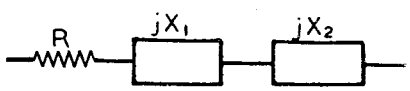


Fig 5

Fig. 5 schematically represents an impedance combination consisting of a resistance R in series with two reactances X_1 and X_2 where either X_1 or X_2 may be a reactance due to inductance or to capacity.

In general terms,

$$Z = R + j(X_1 + X_2)$$

In applying the above expression to any specific case, due regard must be given to the signs of the reactances, using a positive sign with any reactance value due to inductance and a negative sign with any reactance value due to capacity.

Thus, with reference to Fig. 5, if R is a resistance of 200 ohms, X_1 a reactance of 300 ohms due to an inductance and X_2 a reactance of 450 ohms due to a condenser,

$$Z = 200 + j\,(300 - 450)$$
$$= 200 - j\,150 \text{ ohms}$$

Impedance expressions stated in general terms appear frequently in the text of this Instruction and *attention to signs must be given whenever such expressions are applied in calculations relating to specific cases.*

Series and Parallel Impedance Combinations

In order to avoid possible confusion between series and parallel impedance expressions, when given in complex form, the sign // is used to distinguish the parallel cases.

Thus 24+j200 ohms refers to an impedance combination of 24 ohms resistance in series with 200 ohms inductive reactance. 24//j200 ohms refers to an impedance combination comprising 24 ohms resistance in parallel with 200 ohms inductive reactance.

SECTION B. AERIAL DRIVING POINT IMPEDANCE.

The design of an A.T.H. network is based primarily upon :—

(*a*) The impedance of the aerial (looking into it at the driving point) at the carrier frequency.

(*b*) The characteristic impedance of the feeder.

In practice the aerial impedance is determined by bridge measurement. Under ideal conditions, when the dimensions of the aerial and the operating frequency alone control the impedance, the latter might be calculated to a satisfactory degree of accuracy, but in practice the aerial impedance is influenced by masts, stays, other aerials, etc., on the site, and measurement is always necessary. The influence that neighbouring conductors have upon the impedance of an aerial depends upon a number of factors in addition to relative distances and orientations, e.g. resonant frequencies, R.F. resistances, etc. When a bridge measurement of the driving-point impedance of an aerial is being made care should be taken to ensure that all other aerials on the site are terminated correctly and that metal masts are in their normal condition as regards earthing or insulation.

Over the medium-wave frequency band the driving-point impedance of an aerial goes through considerable changes as regards both the resistive and the reactive components. The reactance will be zero at frequencies corresponding with quarter-wave and half-wave operation. At frequencies lower than that corresponding with quarter-wave operation the reactance will be negative, at frequencies between the quarter-wave

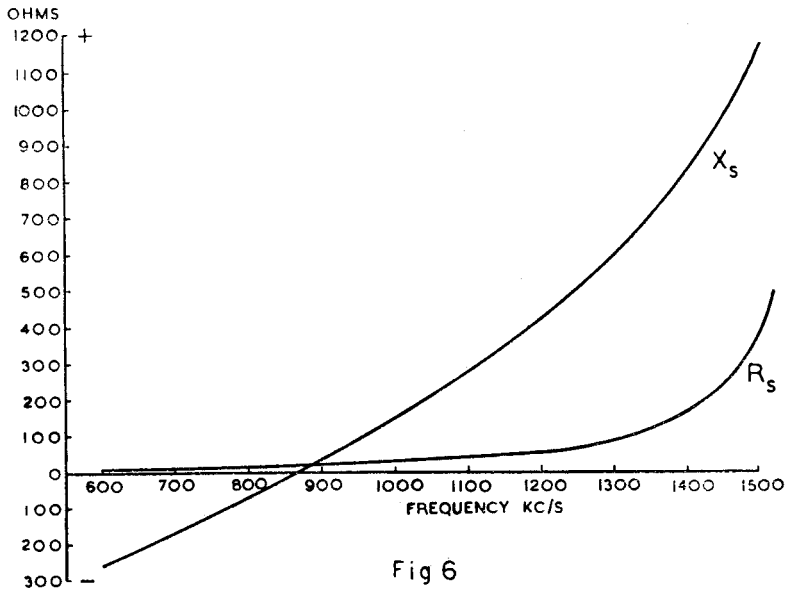


Fig 6

and half-wave conditions it will be positive, becoming negative again between half-wave and three-quarter wave.

The resistive component of the aerial impedance is a value which includes the radiation resistance and the loss resistances. As far as the driving-point impedance of the aerial is concerned, it should be noted that the resistive component is low under quarter-wave conditions and high under half-wave conditions.

When bridge measurements are taken of the driving-point impedances of aerials on BBC sites, it is usual to make a frequency run and to plot the impedance characteristic over the wave-band. Figs. 6 and 7 are two examples.

Fig. 6 is plotted from the results of bridge measurements of Z_s taken on a certain small sloping single-wire aerial. The length of the top portion is 150 ft. and the average height 75 ft. A single-wire downlead, 150 ft. in length, is attached to the centre of the top portion. It will be seen from the figure that quarter-wave conditions occur at a frequency of 870 kc/s, the reactance being zero and the resistance low at this particular frequency.

The curves of Fig. 7 show the impedance char-

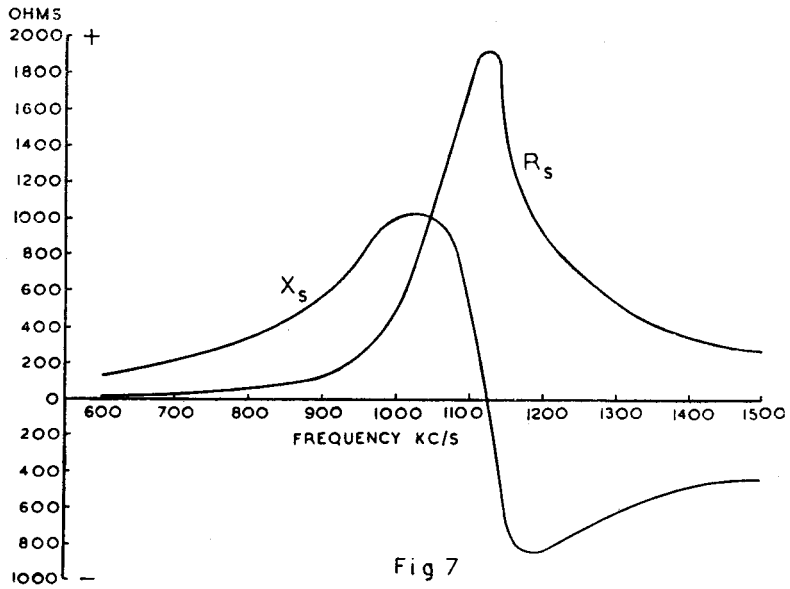


Fig 7

acteristics of a certain large aerial of T type. The top span is of 4-wire construction with a length of 417 ft., while the downlead consists of a vertical 9-wire cage 160 ft. in length. It will be seen from the figure that half-wave conditions occur at a frequency of 1,125 kc/s, the reactance being zero and the resistance high at this frequency.

Figs. 6 and 7 are reproduced for illustrative purposes and mainly with the object of giving emphasis to the fact that the driving-point impedance of any aerial varies very considerably over the medium-wave band.

SECTION C. R.F. FEEDERS.

In the common circumstance where the aerial is situated at such a distance from the transmitter building that the aerial downlead cannot be brought direct to the transmitter, the R.F. power is conveyed from the transmitter to the aerial by a feeder line.

An R.F. feeder must necessarily introduce a certain amount of attenuation due to resistance and insulation leakage, and to radiation. In the following description, however, the assumption will be made that the feeder is dissipationless. This will not be strictly correct in practice, but where feeders used by BBC stations are concerned, usually the losses on medium and long waves due to causes mentioned above are negligible and will, therefore, not be considered in this Instruction.

Characteristic Impedance.

Suppose a source of R.F. power is connected to one end of a long feeder, the other end of which is open. Since the feeder is assumed to be dissipationless and there is no terminating load, there will be nothing to absorb the power. Under these conditions power arriving at the open end will be reflected back along the feeder, returning to the source. At any point between the source and the open end of the feeder there will thus be waves moving forward from the source and also waves moving back towards the source. These forward and backward waves will interact in such a manner that *standing waves* are produced on the feeder, i.e. stationary current and voltage nodal (minimum) and antinodal (maximum) points will be set up along the feeder. The voltage nodal points will be spaced at half-wavelength distances and the antinodal points will also be spaced at half-wavelength distances, while there will be a quarter-wavelength distance between any node and an adjacent antinode. As there is no power absorption there will be 90° phase difference between the current and voltage fluctuations at any point on the feeder. Thus a current node will coincide with a voltage antinode and a voltage node will coincide with a current antinode.

In the case under consideration there will be a current node and a voltage antinode at the open end of the feeder. Suppose that, instead of the feeder being open ended it is short circuited. Again, there can be no power absorption and standing waves will be produced on the feeder. In this case the short-circuited end of the feeder will be a point at which there will be a current antinode and a voltage node.

If a resistance load is connected across the end of the feeder remote from the source, there will be absorption of power in this load, but it will be found that the resistance value of the load must be of a certain critical value if there is to be no reflection. If the load resistance is greater or smaller than this critical value, there will be part absorption and part reflection of power, the reflected component setting up standing waves on the feeder.

The value of terminating resistance, which will give complete absorption of power and no standing waves on the feeder, is equal to the *characteristic impedance* of the feeder.

The characteristic impedance of a dissipationless feeder is a non-reactive resistance value of :

$$\sqrt{\frac{L}{C}} \text{ ohms}$$

where L = inductance per unit length of feeder

C = capacity ,, ,, ,, ,, ,,

The derivation of this formula will be found in a number of standard text books.

It will be seen therefore that the characteristic impedance of a dissipationless feeder will be dependent upon the cross-section and spacing of the conductors and upon the conductor configuration, but it will *not* depend upon the length of the feeder.

Formulae are given in Appendix C, page 55, which enable approximate calculation to be made of the characteristic impedances of certain types of R.F. feeder. Such differences as are found between calculated and measured values are due to factors, dependent upon the particular physical construction employed, such as effects of supporting insulators, capacity to earth (in the case of open wire feeders), etc.

Measurement of Characteristic Impedance

The characteristic impedance of a feeder line can be measured as follows :—

A convenient length of feeder should first be short-circuited at one end, while a measurement of the impedance (series aspect) looking into the feeder, is made by means of an R.F. bridge connected to the other end. Next a measurement should be made with the end of the feeder remote from the bridge open-circuited.

Then :—

$$Z_o = \sqrt{Z_{sc}\, Z_{oc}}$$

where Z_o = characteristic impedance of feeder

Z_{sc} = bridge measurement with far end of feeder short-circuited.

Z_{oc} = bridge measurement with far end of feeder open-circuited.

Note: The symbol R_o will be used for characteristic impedance in diagrams and examples in this Instruction because in all cases the characteristic impedance of a feeder will be assumed to be purely resistive.

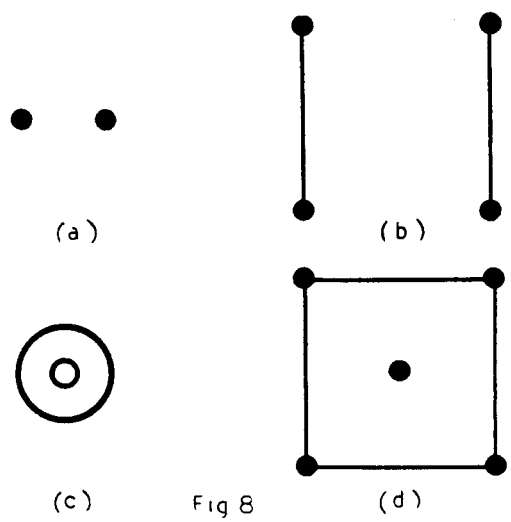


Fig 8

Types of R.F. Feeder.

The various types of feeders may be broadly classified into two groups: (i) Balanced with respect to earth, (ii) Unbalanced with respect to earth.

Fig. 8 shows cross-sectional sketches of certain feeders in common use at BBC stations.

Sketch (a) illustrates the 2-wire balanced feeder. Using 19/.064 wire with a spacing of 18 in. between the two conductors, the measured characteristic impedance of a certain feeder of this type is 550 ohms. The calculated value, using formula (30c), Appendix C, is 566 ohms.

Sketch (b) illustrates a 4-wire feeder with equal horizontal and vertical spacing and with the wires parallelled in two vertical pairs. A certain feeder of this type constructed of No. 6 S.W.G. wire with 12 in. spacing has a measured characteristic impedance of 320 ohms. The calculated value, using formula (31C), Appendix C, is 310 ohms.

Sketch (c) illustrates the concentric feeder. This feeder is of unbalanced type, the outer conductor being earthed. At the pre-war Regional/National transmitting stations concentric feeders were installed, employing an inner copper tubular conductor of 1.375 in. outside diameter and an outer tubular conductor of 5 in. inside diameter. The measured characteristic impedance is 80 ohms. The calculated value, using formula (32C), Appendix C, is 77 ohms.

Sketch (d) illustrates the 5-wire unbalanced feeder. With this type of feeder the four outer wires are parallelled and earthed. A certain feeder of this type employing four outer conductors of No. 6 S.W.G. wire spaced 12 in. horizontally and vertically and with a central inner conductor of 19/.044 wire has a measured characteristic impedance of 300 ohms. Using formula (33C), Appendix C, and assuming all conductors of the diameter of 19/.044 wire, gives a calculated value of 310 ohms. The fact that in this particular case there is a slight difference between the diameters of the inner and the outer conductors can be disregarded in view of the fact that the calculation by formula represents an approximation only.

Matching.

In view of the information given in Sections B and C, it is now possible to amplify the information given in Section A regarding the matching network in an aerial transformer house.

The two impedances which have to be matched by the network are the aerial impedance on the one hand and the characteristic impedance of the feeder on the other. This implies that the combination of aerial and network must in the ideal case present to the feeder a load of Z_o ohms, where Z_o is the characteristic impedance of the feeder.

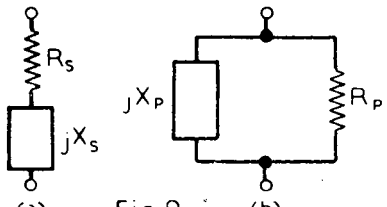
Except when the aerial is being operated under quarter-wave or half-wave conditions the aerial impedance will be complex. (The term complex is used here in its mathematical sense and does not mean " complicated.") The characteristic impedance of the feeder, however, will be non-reactive (assuming a dissipationless feeder). It will be

seen, therefore, that one possibility regarding an A.T.H. network is that it consists of (a) reactive elements, the purpose of which is to balance out the reactive component of the aerial impedance and (b) reactive elements, the purpose of which is to convert the resulting non-reactive resistance to a value equal to the characteristic impedance of the feeder. The network used to match the impedance of the aerial to the characteristic impedance of the feeder is often called a *transducer*.

A complete A.T.H. network, however, cannot always be regarded in such a simple manner as described above, i.e. regarded as made up of two separate parts, one balancing out aerial reactance and the other providing the necessary resistance transformation, although this sometimes applies. In some cases, for example, the whole or part of the aerial reactance is incorporated in the transducer and when this is possible the theoretical considerations are a little more complicated, although the practical set-up may be greatly simplified and costs reduced. This matter is considered in detail in Section H (A.T.H. circuits). As a preliminary, however, the theory of transducers will be dealt with without reference to aerial reactance.

SECTION D. SERIES AND PARALLEL IMPEDANCE EQUIVALENTS.

Any impedance represented by a resistance in series with a reactance is equivalent, at any given single frequency, to a particular parallel combination of resistance and reactance. The converse also applies.



(a) Fig 9 (b)

In Fig. 9 (a) is shown a resistance, R_s, in series with a reactance, X_s, the latter being shown in block schematic form, since the argument applies equally well whether the reactance be positive or negative, provided that the reactance sign is the same in both the series and the parallel cases. In Fig. 9 (b) is shown a resistance, R_p, in parallel with a reactance, X_p. The two impedance combinations of Figs. 9 (a) and 9 (b), respectively, can be made equivalent to each other, at any given single frequency, provided that certain relations of values are established, the correct relations being derived later in this section.

Assuming that the correct relation of values exists, it follows from the statements made above that the series combination of Fig. 9 (a) could be replaced by the parallel combination of Fig. 9 (b) and vice versa. In point of fact, if two series and parallel equivalent impedances were individually made up and separately boxed without distinguishing marks, it would be impossible by an R.F. bridge measurement of impedance (made at the frequency of equivalence) to determine which box contained the series combination and which the parallel combination.

It must be emphasized that the equivalence holds good only at the one particular frequency.

A far-reaching consequence of the series-parallel equivalent impedance rule is that, even though an impedance may actually consist of elements connected in series, it is perfectly legitimate to treat it as though it existed in the form of its parallel equivalent, and the converse holds good.

Series to Parallel Equivalent Impedance Conversion.

The parallel combination of Fig. 9 (b) will be equivalent to the series combination of Fig. 9 (a) if,

$$R_p = \frac{R_s^2 + X_s^2}{R_s} \quad \text{.....(3)}$$

and

$$X_p = \frac{R_s^2 + X_s^2}{X_s} \quad \text{.....(4)}$$

the sign of X_p being the same as the sign of X_s.

For the purpose of slide-rule calculation the expressions for R_p and X_p are more convenient if given in the following form :—

$$R_p = R_s\left(1 + \frac{X_s^2}{R_s^2}\right) \quad \text{.....(5)}$$

and

$$X_p = X_s\left(1 + \frac{R_s^2}{X_s^2}\right) \quad \text{.....(6)}$$

The derivation of the above formulae is given in Appendix A, page 47.

Example 1: What is the parallel equivalent of $38-j76$ ohms?

Answer:

From equation (3),

$$R_p=38\left(1+\frac{(-76)^2}{38^2}\right)=38\times 5=190.$$

From equation (4),

$$X_p=-76\left(1+\frac{38^2}{(-76)^2}\right)=-76\times 1.25=-95$$

∴ Parallel equivalent of $38-j76$ ohms is $190//-j95$ ohms.

A fact that can be applied very usefully for numerical checking purposes is that the product of the series and parallel equivalent resistance values is equal to the product of the series and parallel equivalent reactance values.

$$\text{i.e. } R_pR_s=X_pX_s$$

That this is true can be seen by inspection of equations (3) and (4).

From equation (3), $R_pR_s=R_s^2+X_s^2$

From equation (4), $X_pX_s=R_s^2+X_s^2$

$$\therefore\ R_pR_s=X_pX_s \quad \text{.....(7)}$$

In example 1, $R_pR_s=190\times 38=7220$, while $X_pX_s=-95\times -76=7220$.

Alternative Values of X_s for Given Value of X_p, R_s being Constant.

A case of some practical significance is that in which an impedance R_s+jX_s has a constant value of R_s, but X_s is variable and of either positive or negative sign. It can be shown that in these circumstances X_s may in general be given either of two alternative values which will give rise to the same value of equivalent parallel reactance X_p, which will always be of the same sign as X_s. The resulting values of equivalent parallel resistance R_p will, of course, be different in the two cases. These two values of X_s are given by :—

$$X_{s_1}=\frac{X_p}{2}+\sqrt{\left(\frac{X_p}{2}+R_s\right)\left(\frac{X_p}{2}-R_s\right)} \quad \text{.....(8)}$$

and

$$X_{s_2}=\frac{X_p}{2}-\sqrt{\left(\frac{X_p}{2}+R_s\right)\left(\frac{X_p}{2}-R_s\right)} \quad \text{.....(9)}$$

If, however, X_p is numerically equal to $2R_s$, the two values of X_s become identical and numerically equal to R_s. If X_p is numerically less than $2R_s$, no real value of X_s can be found to satisfy the required condition, because the quantity under the square root sign becomes negative.

The derivation of the above formulae is given in Appendix A, page 47.

Example 2: If $Z_s=124+jX_s$ ohms, what values of X_s will give an equivalent parallel reactance of 520 ohms?

Answer:

From equations (8) and (9)

$$X_s=260\pm\sqrt{384\times 136}$$
$$=260\pm\sqrt{52{,}220}$$
$$=260\pm 228.5$$
$$\therefore\ X_s=488.5 \text{ or } 31.5 \text{ ohms.}$$

Of the two values of X_s, which give the same value of X_p (R_s being constant), one is greater than R_s, while the other is smaller. Further, if the larger value is n times R_s then the smaller value will be 1/nth of R_s.

Example 2 illustrates this fact for 488.5 is equal to 124×3.94, while 31.5 is equal to 124/3.94.

That both nR_s and R_s/n are values of X_s giving the same value of X_p can be proved by substituting first $X_s=nR_s$ and then $X_s=R_s/n$ in equation (4). Substitution in equation (3) gives the two resulting values of R_p.

Parallel to Series Equivalent Impedance Conversion.

The series combination of Fig. 9 (a) will be equivalent to the parallel combination of Fig. 9 (b) if

$$R_s=\frac{R_pX_p^2}{R_p^2+X_p^2} \quad \text{.....(10)}$$

and

$$X_s=\frac{R_p^2X_p}{R_p^2+X_p^2} \quad \text{.....(11}$$

the sign of X_s being the same as the sign of X_p.

For the purposes of slide-rule calculation the expressions for R_s and X_s are more convenient if given in the following form :—

$$R_s=\frac{R_p}{1+R_p^2/X_p^2} \quad \text{.....(12)}$$

and

$$X_s = \frac{X_p}{1 + X_p^2/R_p^2} \quad \ldots\ldots (13)$$

The derivation of the above formulae is given in Appendix A, page 47.

Example 3: What is the series equivalent of 240//j85 ohms?

Answer:

From equation (12),

$$R_s = \frac{240}{1 + 240^2/85^2} = \frac{240}{1 + 7.98}$$

$$= \frac{240}{8.98} = 26.7 \text{ ohms}$$

From equation (13),

$$X_s = \frac{85}{1 + 85^2/240^2} = \frac{85}{1 + 0.126}$$

$$= \frac{85}{1.126} = 75.5 \text{ ohms}$$

∴ Series equivalent of 240//j85 ohms is 26.7 + j75.5 ohms.

It has already been shown (page 10), with the aid of equations (3) and (4), that $R_pR_s = X_pX_s$. Inspection of equations (10) and (11) again shows that $R_pR_s = X_pX_s$. It is to be noted, however, that equations (10) and (11) do not provide a proof of the statement independently of equations (3) and (4), since the former pair of equations are derived from the same original equation as are the latter.

Alternative Values of X_p for Given Value of X_s, R_p being Constant.

In the case where an impedance $R_p//jX_p$ has a constant value of R_p, but X_p is variable and of either positive or negative sign, it can be shown that in general the same value of equivalent series reactance, X_s, is produced when X_p is given either of two alternative values. The resulting values of equivalent series resistance R_s will, of course, be different in the two cases. These two values of X_p are given by:—

$$X_{p1} = \frac{R_p}{2X_s}\left[R_p + \sqrt{(R_p + 2X_s)\,(R_p - 2X_s)}\right] \ldots (14)$$

$$X_{p2} = \frac{R_p}{2X_s}\left[R_p - \sqrt{(R_p + 2X_s)\,(R_p - 2X_s)}\right] \ldots (15)$$

If, however, X_s is numerically equal to $\frac{1}{2}R_p$, the two values of X_p become identical and numerically equal to R_p. If X_s is numerically greater than $\frac{1}{2}R_p$, no real value of X_p can be found to satisfy the required condition because the quantity under the square root sign becomes negative.

The derivation of the above formulae is given in Appendix A, page 47.

Example 4: If $Z_p = 124//jX_p$ ohms, what values of X_p will give an equivalent series reactance of −50 ohms?

Answer:

From equations (14) and (15),

$$X_p = -\frac{124}{100}\left[124 \pm \sqrt{224 \times 24}\right]$$

$$= -1.24\,[124 \pm 73.31]$$

$$\therefore\ X_p = -224.6 \text{ or } -62.87 \text{ ohms.}$$

Example 5: What values of series equivalent resistance are given by the two values of X_p in Example 4?

Answer:

From equation (7),

$$R_s = \frac{X_pX_s}{R_p}$$

$$= \frac{244.6 \times 50}{124} \text{ or } \frac{62.87 \times 50}{124}$$

$$\therefore\ R_s = 98.65 \text{ or } 25.36 \text{ ohms.}$$

The two values of X_p which give the same value of series equivalent reactance are related to R_p in the following manner:—

If one of the X_p values is nR_p, the other will be R_p/n. Proof can be obtained by substituting, first $X_p = nR_p$ and then $X_p = R_p/n$ in equation (11).

SECTION E. L-TYPE TRANSFORMING NETWORKS.

Transforming Downwards.

The first case to be considered will be that where it is required to transform a resistance R_A to a *smaller* resistance value R_B. The upwards transforming case will be dealt with afterwards.

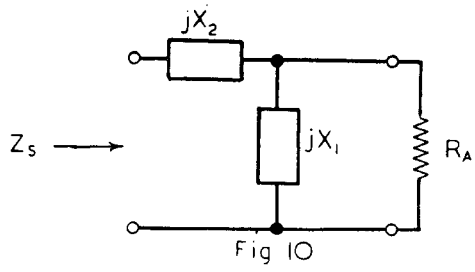


Fig 10

Suppose a reactance X_1 is connected in shunt with R_A, as in Fig. 10, and let the series equivalent impedance of R_A and X_1 in parallel be R_s+jX_s.

R_s must necessarily be smaller than R_A and, by suitable choice of X_1, can be made any value between R_A and zero. Let the value of X_1 be chosen so that $R_s=R_B$. The series equivalent impedance of R_A and X_1 in parallel is then R_B+jX_s.

If a reactance X_2 is connected in series with the parallel combination of R_A and X_1, then the series equivalent impedance looking into the terminals of the network is,

$$Z_s=R_B+j(X_s+X_2)$$

If X_2 is made equal in value and opposite in sign to X_s, that is $X_2=-X_s$, then,

$$Z_s=R_B$$

which is the desired condition.

The L network composed of the reactances X_1 and X_2 thus transforms R_A down to R_B.

It will be seen that a downwards L transducer consists of (i) a shunt reactive element, the value of which is such as to make the equivalent series resistance equal to the required value of transformed resistance, and (ii) a series reactive element of opposite sign to the shunt element and equal in value to the equivalent series reactance.

Since the shunt and series elements must be of opposite sign, there are two alternative basic forms of the downwards L transducer, as shown in Figs. 11 (a) and 11 (b).

Where A.T.H. circuits are concerned, preference is generally given to the network of Fig. 11 (a). To have capacity in the shunt arm and inductance in the series arm gives a network providing attenuation of harmonic frequencies of the carrier.

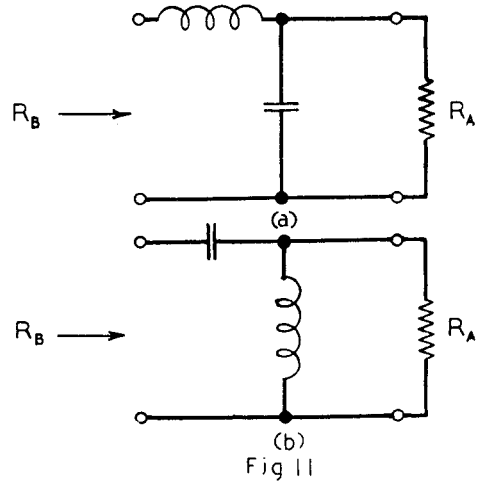


Fig 11

This is important if the output circuits of the transmitter do not themselves reduce the harmonic content of the carrier output to negligible proportions. Apart from this consideration, which will be of greater or less importance according to circumstances, the network of Fig. 11 (a) avoids the use of a series condenser.

The installation of a series condenser inevitably introduces capacity to earth. High power condensers have large cases often connected to one side of the condenser—and it is difficult to prevent the capacity to earth being sufficiently large to modify appreciably the performance of the network and in some cases it may prove awkward to obtain the required final result. In addition, large capacity to earth may give heavy losses in surrounding brickwork, etc., unless screening is employed.

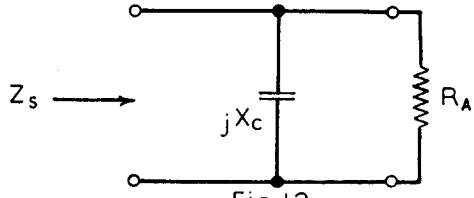


Fig 12

Dealing more specifically with downwards L transducers of the form shown in Fig. 11 (a), consider a case in which the shunt condenser is connected but the series inductance is omitted, as in Fig. 12.

From equations (10) and (11), page 10,

$$Z_s=\frac{R_AX_C^2}{R_A^2+X_C^2}+j\frac{R_A^2X_C}{R_A^2+X_C^2} \quad \text{.....(16)}$$

Let the capacity be so chosen that,

$$\frac{R_A X_C^2}{R_A^2+X_C^2}=R_B$$

then,

$$Z_s=R_B+j\frac{R_A^2 X_C}{R_A^2+X_C^2} \quad \text{.....(17)}$$

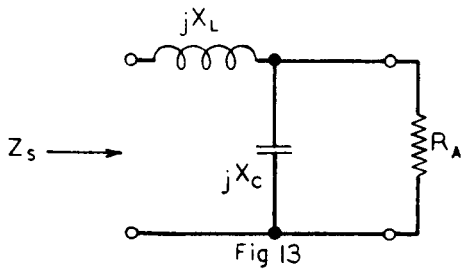


Fig 13

If a series inductance is now connected as shown in Fig. 13,

$$Z_s=R_B+j\left(X_L+\frac{R_A^2 X_C}{R_A^2+X_C^2}\right) \quad \text{.....(18)}$$

If the inductance is so chosen that,

$$X_L=-\frac{R_A^2 X_C}{R_A^2+X_C^2}$$

then,

$$Z_s=R_B$$

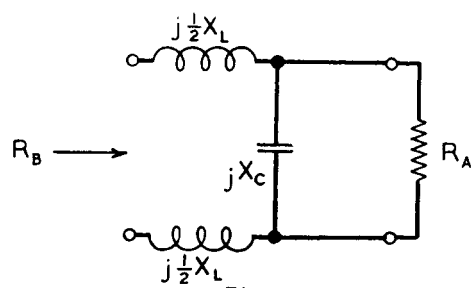


Fig 14

The circuit of Fig. 13 is of unbalanced type. Should it be required that the circuit shall be balanced the total inductance can be split into two halves, connected as shown in Fig. 14.

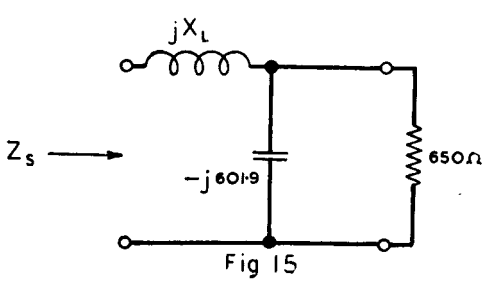


Fig 15

Example 6: What are the values of Z_s in Fig. 15 when X_L has the following values :—

(*a*) +210 ohms
(*b*) +400 ohms
(*c*) +324.1 ohms

Answer:

From equation (12) the equivalent series resistance of 650//−j601.9 ohms is,

$$\frac{650}{1+650^2/(-601.9)^2}=\frac{650}{2.166}=300 \text{ ohms}$$

From equation (13) the equivalent series reactance of 650//−j601.9 ohms is,

$$\frac{-601.9}{1+(-601.9)^2/650^2}=\frac{-601.9}{1.857}=-324.1 \text{ ohms}$$

Thus, in case (*a*) $Z_s=300-j324.1+j210$
$=300-j114.1$ ohms

In case (*b*) $Z_s=300-j324.1+j400$
$=300+j75.9$ ohms

In case (*c*) $Z_s=300-j324.1+j324.1$
$=300+j0$ ohms

Case (*c*) of example 6 is a case where there is a transformation from 650 ohms resistance to 300 ohms resistance. In case (*a*) the inductance is not large enough to balance out the equivalent series reactance of the condenser and 650 ohms resistance in parallel, while in case (*b*) it is too large and Z_s has consequently a component of positive reactance.

It will be noted that the assumption is made in example 6 that the equivalent series resistance, 300 ohms, does not change with variation of the inductance. This would only be the case if the inductance had no R.F. resistance and if stray capacities between components and from components to earth did not exist. However, with a low-loss coil carefully mounted to minimise strays the change of R_s will be very slight. As approximate calculations only are dealt with in this Instruction, the results given in example 6 are permissible.

Design Formulae. Downwards L Transducer.

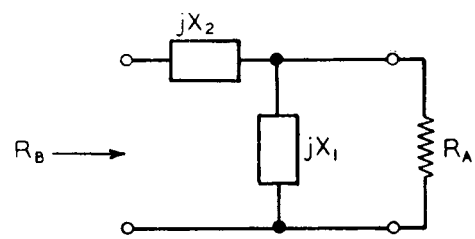


Fig 16

Fig. 16 shows an L-type transducer which is transforming the resistance value R_A down to a lower value R_B. The required values for X_1 and

X_2 can be calculated by the following simple formulae :—

$$X_1 = \pm\frac{mR_B}{\sqrt{m-1}} \quad \text{.....(19)}$$

$$X_2 = \mp R_B\sqrt{m-1} \quad \text{.....(20)}$$

where $m = \frac{R_A}{R_B}$

The derivation of the above formulae is given in Appendix A, page 48.

Example 7 : What are the reactance values for an L-type transducer of the form illustrated in Fig. 11 (a), which will transform a resistance of 550 ohms to a resistance of 80 ohms ?

Answer :

$$m = \frac{550}{80} = 6.875$$

$$\sqrt{m-1} = 2.424$$

From equation (20),

$$X_L = +80 \times 2.424$$
$$= +193.9 \text{ ohms}$$

From equation (19),

$$X_C = -\frac{6.875 \times 80}{2.424}$$
$$= -227 \text{ ohms}$$

Example 8 : Calculate the L and C values and draw the circuit of a transducer of the form illustrated in Fig. 11 (a), which will transform a resistance of 1900 ohms to a resistance of 550 ohms at a frequency of 668 kc/s.

Answer :

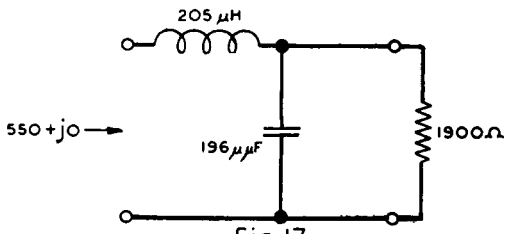


Fig 17

$$m = \frac{1900}{550} = 3.45$$

$$\sqrt{m-1} = 1.565$$

From equation (20),

$$X_L = +550 \times 1.565$$
$$= +860.8 \text{ ohms}$$

$$\therefore\ L = \frac{860.8 \times 10^3}{2\pi \times 668} = 205\mu H$$

From equation (19),

$$X_C = -\frac{3.45 \times 550}{1.565}$$
$$= -1213 \text{ ohms}$$

$$\therefore\ C = \frac{10^9}{2\pi \times 668 \times 1213} = 196\mu\mu F$$

The diagram is given in Fig. 17.

Example 9 : If a 300 $\mu\mu$F condenser were substituted for the condenser shown in Fig. 17, what change of inductance would be required to restore unity power factor (looking into the terminals of the network), and what change would occur in the transformed resistance value ? Assume that the frequency remains at 668 kc/s.

Answer :

The reactance of 300 $\mu\mu$F at 668 kc/s is,

$$X_C = -\frac{10^9}{2\pi \times 668 \times 300} = -793.4 \text{ ohms}$$

From equation (13) the equivalent series reactance of 1900//−j793.4 ohms is,

$$X_s = \frac{-793.4}{1 + (-793.4)^2/1900^2} = -675.7 \text{ ohms}$$

For unity power factor at the input terminals of the network it will be necessary to adjust the inductance to provide a reactance of +675.7 ohms.

At 668 kc/s this will require an inductance value of,

$$L = \frac{675.7 \times 10^3}{2\pi \times 668} = 160.7\ \mu H$$

Thus the inductance must be reduced from 205 μH to 160.7 μH.

From equation (12), the equivalent series resistance of 1900//−j793.4 ohms is,

$$R_s = \frac{1900}{1 + 1900^2/(-793.4)^2} = 282 \text{ ohms}$$

Thus the value of transformed resistance would drop from 550 ohms to 282 ohms.

The Practical Setting-up of a Downwards L Transducer.

The performance of a transducer may be found in practice to differ somewhat from that assumed in the " paper design " for the reason that the design formulae given in this Instruction take no account of stray capacities, inductances and couplings, nor of R.F. resistance and dielectric losses.

The effects of " strays " can be considerable and it is necessary to construct the network with care, adopting as far as circumstances will permit all the usual methods, familiar to transmitter engineers, of keeping " strays " at a minimum. In particular, capacities from coils to earth should be kept down as low as possible.

Fixed condensers are frequently used as the shunt elements in L type networks. The nearest capacity value available may in some cases differ from the required value sufficiently to prevent the desired impedance match being obtained without modification of the network. A method of overcoming this difficulty is as follows :—

If the condenser introduces too much negative reactance (i.e. if the capacity is too small), an inductance should be connected in series with it and adjusted to a value which cancels the excess negative reactance. In a normal case only a small value of inductance will be required.

If the reactance of the condenser is too small (i.e. if the capacity is too large), the required value of negative reactance can be secured by the use of a suitable value of inductance connected in shunt with the condenser. In a normal case the inductance value required will be comparatively large and this artifice is consequently not so convenient as that described for the smaller capacity case.

The adjustments that are required in setting up a terminated transducer to give the required value of transformed resistance are best made with the aid of an R.F. bridge.

It should be noted that the standard BBC bridge is one that provides impedance measurements in *parallel terms*. Thus, if the bridge is used to measure an impedance combination made up of resistance and reactance connected in series the direct results of the bridge measurement will give the equivalent parallel resistance and reactance of the series combination. In cases where the series values are required, these can be obtained from the measured equivalent parallel values by simple calculation, as described in Section D of this Instruction.

The practical adjusting of a downwards L transducer, terminated by a resistance R_A, to give the required value of transformed resistance will be considered in relation to two possible cases :—

Case 1.—Network of the form shown in Fig. 11 (a). Shunt capacity fixed. Series inductance variable.

Case 2.—Network of the form shown in Fig. 11 (a). Shunt capacity variable. Series inductance variable.

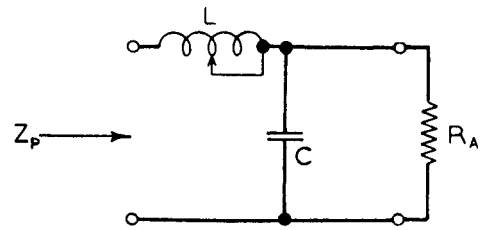


Fig 18

Case 1. – Shunt capacity fixed. Series inductance variable. Fig. 18.

With the bridge connected to measure Z_P, Fig. 18, adjust L until unity power factor is obtained at the input terminals of the network. That is, adjust L until the bridge indicates that the reactive component of Z_P is infinite. In this connection it must be remembered that $R//j\infty$ is equivalent to $R+j0$.

The resistance value as measured by the bridge is the value of the transformed resistance.

If it is found that there is a greater difference than is permissible between the expected and measured values of transformed resistance a check should be made upon the equivalent series resistance component of $R_A//jX_C$. To do this, disconnect L from C and connect the bridge directly across C. From the bridge measurements calculate the equivalent series resistance value.

If L has negligible influence upon the transformed resistance value the series resistance, determined as above, will be equal to the transformed resistance as previously measured. If this proves to be the case then an incorrect value of C should be presumed. If, however, the test shows that L is responsible for the incorrect value of transformed resistance then the existence of excessive " strays " associated with L should be suspected.

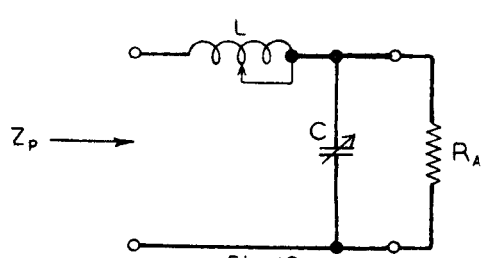


Fig 19

Case 2.—Shunt capacity variable. Series inductance variable. Fig. 19.

Assuming that " strays " have been minimised by careful construction of the circuit, the correct

adjustments of C and L can be made as follows :—

Adjust the condenser to a setting that is considered to be correct. With the bridge connected to measure Z_p, Fig. 19, adjust L for unity power factor at the input terminals of the network.

If the condenser setting approximates to the correct value, the measured value of transformed resistance should now approximate to the required value. Fine adjustment of the transformed resistance value can then be made by fine adjustment of C, re-adjusting L for unity power factor with each change of C.

Transforming Upwards.

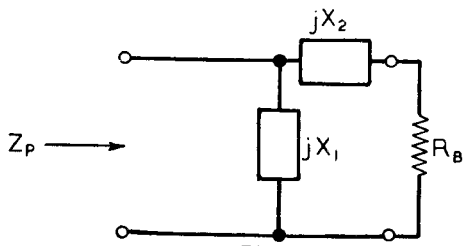


Fig 20

Consideration will now be given to the case where it is required to transform a resistance R_B to a *greater* resistance value, R_A.

Let a reactance X_2 be connected in series with R_B as in Fig. 20, and let the equivalent parallel impedance of R_B and X_2 in series be $R_p//jX_p$. R_p will be greater than R_B and can be made equal to R_A by suitable choice of X_2.

If a shunt reactance, X_1, is connected as shown in Fig. 20, the equivalent parallel impedance, looking into the terminals of the network, is

$$Z_p = R_A//j\left(\frac{X_1\ X_p}{X_1 + X_p}\right) \quad \text{.....(21)}$$

If X_1 is a reactance of opposite sign to X_2 (and therefore to X_p) and is so adjusted that $X_1 = -X_p$, then the expression in brackets becomes infinity. Thus Z_p is equivalent to a resistance R_A shunted by infinite reactance. In other words,

$$Z_p = R_A$$

which is the desired condition.

Thus, an upwards L transducer consists of a series reactive element, the value of which is such as to make the equivalent parallel resistance equal to the required value of transformed resistance, and (ii) a shunt reactive element of opposite sign to the series element and equal in value to the equivalent parallel reactance.

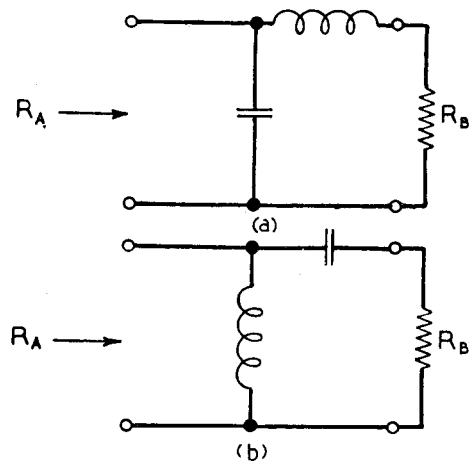


Fig 21

The two basic forms of the upwards L transducer are shown in Figs. 21 (a) and 21 (b). The more commonly used circuit is that of Fig. 21 (a), but in an A.T.H. network that is not required to contribute to the attenuation of harmonics the circuit of Fig. 21 (b) may be used.

It does not necessarily follow where A.T.H. circuits are concerned that a basic schematic of the type of Fig. 21 (b) necessarily implies the use of a series-connected condenser. In some cases the reactive component of the aerial driving-point impedance may provide the required negative reactance without a condenser being necessary.

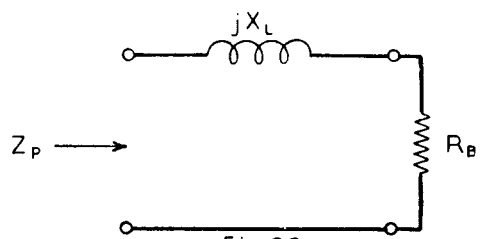


Fig 22

Dealing more specifically with upwards L transducers of the form shown in Fig. 21 (a), consider a case in which the series reactance is connected but the shunt condenser is omitted, as in Fig. 22.

From equations (3) and (4), page 9,

$$Z_p = \frac{R_B^2 + X_L^2}{R_B} //j\left(\frac{R_B^2 + X_L^2}{X_L}\right) \quad \text{.....(22)}$$

The equivalent parallel resistance $\dfrac{R_B^2 + X_L^2}{R_B}$ will be greater than R_B and can be made of any required value (above R_B) by suitable adjustment of X_L. Let the value of X_L be such that the equivalent parallel resistance is R_A.

Then,

$$Z_p = R_A // j\frac{R_B^2 + X_L^2}{X_L} \quad \text{.....(23)}$$

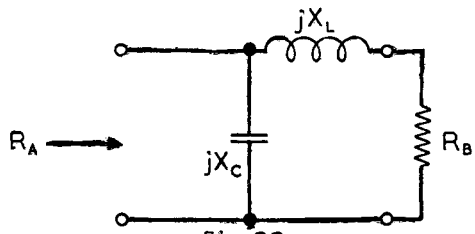


Fig 23

If a shunt condenser is now connected as shown in Fig. 23,

$$Z_p = R_A // j\left(\frac{X\,X_C}{X + X_C}\right) \quad \text{.....(24)}$$

where $X = \frac{R_B^2 + X_L^2}{X_L}$

If X_C is made equal to $-\left(\frac{R_B^2 + X_L^2}{X_L}\right)$ then,

$$Z_p = R_A$$

Design Formulae. Upwards L Transducer.

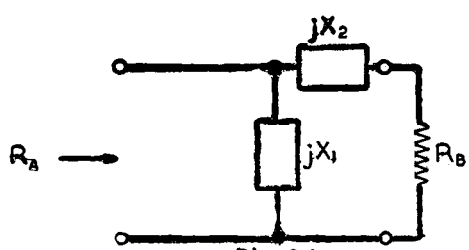


Fig 24

Fig. 24 shows an L-type transducer which is transforming the resistance value of R_B up to a higher value R_A. The required values for X_1 and X_2 can be calculated by the following formulae :—

$$X_1 = \pm\frac{mR_B}{\sqrt{m-1}} \quad \text{.....(25)}$$

$$X_2 = \mp R_B\sqrt{m-1} \quad \text{.....(26)}$$

where $m = \frac{R_A}{R_B}$

The derivation of the above formulae is given in Appendix A, page 48.

General Design Formulae. L-type Transducers.

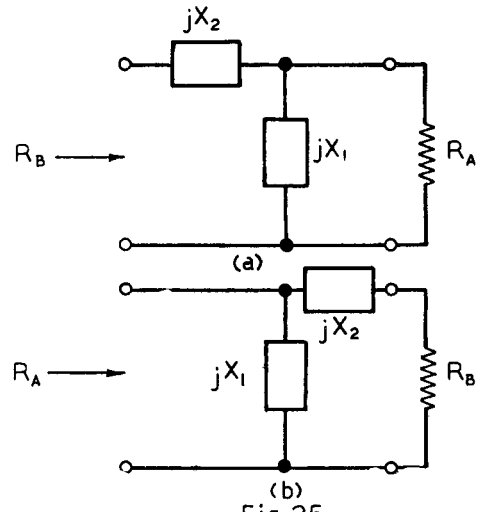


Fig 25

It will have been observed that the formulae

$$X_1 = \pm\frac{mR_B}{\sqrt{m-1}}$$

and

$$X_2 = \mp R_B\sqrt{m-1}$$

where $m = \frac{R_A}{R_B}$

have been evolved for *both* the downwards L and upwards L transducers. The identity of the formulae for the two cases has come about due to the choice of symbols.

It should be particularly noted that in both cases $m = \frac{R_A}{R_B}$, but in the case of the downwards L transducer R_B is the transduced resistance value whereas in the case of the upwards L transducer R_A is the transduced resistance value (see Figs. 25 (a) and 25 (b)).

In treating the above formulae as general design formulae, applicable to either downwards L or upwards L transducers possible confusion will be avoided if the following is noted :—

(i) X_1 is the shunt reactance and X_2 the series reactance.

(ii) m is the ratio of *larger* resistance value to *smaller* resistance value.

Example 10 : Sketch the diagram of an L-type transducer of the type illustrated in Fig. 21 (a), which will transform a resistance of 139 ohms

to a resistance of 300 ohms at 1013 kc/s. Mark in L and C values.

Answer :

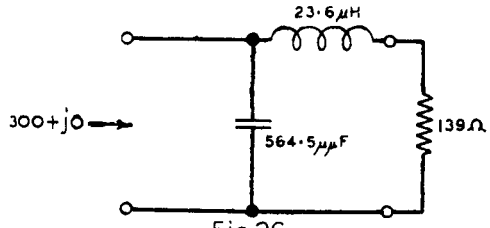


Fig 26

$$M=\frac{300}{139}=2.16$$

$$\sqrt{M-1}=\sqrt{1.16}=1.08$$

$$X_L=+1.08\times 139=+150.1 \text{ ohms}$$

$$X_C=-\frac{2.16\times 139}{1.08}=-278.1 \text{ ohms}$$

$$\therefore\ L=\frac{150.1\times 10^3}{2\pi\times 1013}=23.6\mu H$$

and,

$$C=\frac{10^9}{2\pi\times 1013\times 278.1}=564.5\ \mu\mu F.$$

Example 11 : Referring to Example 10, what is the alternative value of L which will produce unity power factor and what will be the value of transformed resistance corresponding with it?

Answer :

In Example 10 the calculated value of X_L is + 150.1. From the information given on page 10, following Example 2,

$$n=\frac{X_L}{R_s}=\frac{150.1}{139}=1.08$$

It follows that the alternative value of X_L is,

$$\frac{R_s}{n}=\frac{139}{1.08}=+128.7$$

$$\therefore\ L=\frac{128.7\times 10^3}{2\pi\times 1013}=20.2\ \mu H$$

The transformed resistance corresponding to this value of inductance is equal to the equivalent parallel resistance of $139+j128.7$

$$=139\ (1+128.7^2/139^2)$$

$$=258.2 \text{ ohms.}$$

The Practical Setting-up of an Upwards L Transducer.

The general remarks made on page 15 in connection with strays, etc., apply equally well to the case of the upwards L transducer.

The practical adjusting of an upwards L transducer, terminated by a resistance R_B, to give the required value of transformed resistance will be considered in relation to three possible cases :—

Case (1). Network of the form shown in Fig. 21 (a). Series inductance variable. Shunt capacity variable.

Case (2). Network of the form shown in Fig. 21 (a). Series inductance variable. Shunt capacity fixed.

Case (3). Network of the form shown in Fig. 21 (b). Series capacity variable. Shunt inductance variable.

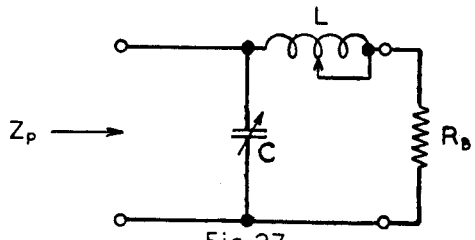


Fig 27

Case (1): Series inductance variable. Shunt capacity variable. Fig. 27.

From the information given on page 16 it will be understood that the correct adjustment of L, Fig. 27, will be such as to make the equivalent parallel resistance component of R_B+jX_L equal to the required value of transformed resistance.

With the bridge connected to measure Z_p, adjust L to provide a resistive component of the measured Z_p equal to the required value of transformed resistance. Then adjust C for unity power factor, i.e. adjust C to the setting that makes the reactance of the measured Z_p infinite.

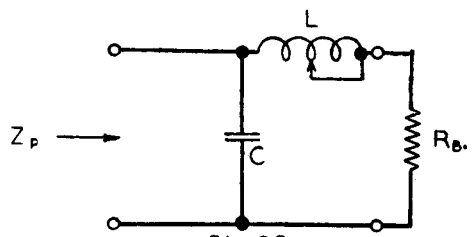


Fig 28

Case (2): Series inductance variable. Shunt capacity fixed. Fig. 28.

This case is the one that has to be dealt with most frequently. The method described above for adjustment of L will not be satisfactory in this case for the reason that the adjustment must now be such as to make the equivalent parallel reactance of R_B+jX_L (Fig. 28) equal and opposite in value

to the reactance of C, in order to secure unity power factor.

In the case of the upwards L transducer the effects of varying the inductance in the series arm of the transducer when the shunt capacity is fixed are more complex than in the case of the downwards L transducer.

With the exception of the case where the transformation ratio is 1 : 2 there will be *two* possible values of inductance, either of which will produce zero phase angle (unity power factor) at the terminals of the transducer. Reference to page 10, equations (8) and (9), will make the reason clear.

Since the value of inductance controls the transformation ratio of the network, the two alternative values of L producing unity power factor *will not produce the same value of transformed resistance.* If this fact is remembered, there should be no risk in practice of adjusting the inductance incorrectly.

When there are two possible values of L which will give unity power factor at the terminals of the network the rule indicating whether the smaller or the larger value will be correct is :—If the required transformed resistance value is less than twice the value of the resistance being transformed, then the smaller of the two L values is correct. If the required transformed resistance value is greater than twice the value of the resistance being transformed, then the larger of the two L values is correct.

With the bridge connected to measure Z_P, Fig. 28, adjust L to the value which gives infinite reactance at the input terminals of the network, carefully checking the transformed resistance value to ensure that the wrong alternative setting of L has not been used.

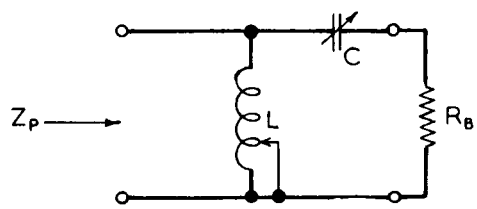


Fig 29

Case (3). Series capacity variable. Shunt inductance variable. Fig. 29.

The schematic of Fig. 29 shows the series arm as consisting of a plain variable condenser, but in practice it is very unlikely to be so. Probably the variable capacitive reactance will be made up of a fixed capacitive reactance (perhaps the reactance of the aerial) in series with a variable inductance. For the moment the simplified equivalent shown in Fig. 29 will suffice.

The series arm of the network should be adjusted first, the correct adjustment being that which makes the resistive component of Z_P equal to the required value of transformed resistance. Then adjust L to give unity power factor.

SECTION F. T-TYPE TRANSFORMING NETWORKS.

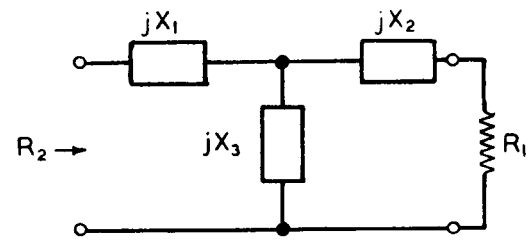


Fig 30

Fig. 30 is a schematic of a transducer of T structure transforming R_1 to R_2. In general, R_2 may be smaller or greater than R_1. It is essential that one of the reactance elements be of opposite sign to the other two.

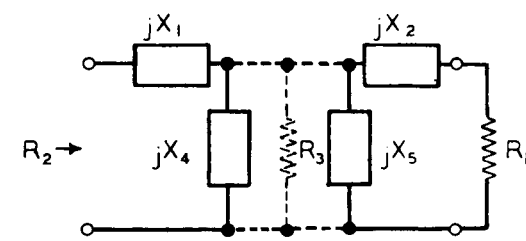


Fig 31

The T network of Fig. 30 may be regarded as equivalent to two L-type transducers connected together, one transforming upwards and the other transforming downwards. Fig. 31 is the equivalent of Fig. 30, if,

$$\frac{X_4X_5}{X_4+X_5}=X_3$$

X_2 and X_5 form an upwards L transducer transforming R_1 up to R_3, where R_3 is the mid-shunt resistance of the T network, while X_4 and X_1 form a downwards L transducer transforming R_3 down to R_2.

T-type transducers are very frequently incorporated in A.T.H. circuits. Using a shunt element of capacitive reactance a T-type transducer is preferable to a simple L-type transducer in any case where the ratio of transformation is near unity and the use of a simple L-type would not provide adequate attenuation of harmonics. Also in cases where the capacitive element is fixed and of an unsuitable value for use in a simple L net-