TECHNICAL INSTRUCTION

TT.5

Medium-Wave Impedance-Matching Networks.

Rejectors and Acceptors.

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BRITISH BROADCASTING CORPORATION

ENGINEERING DIVISION

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CONTENTS

MEDIUM-WAVE IMPEDANCE-MATCHING NETWORKS. REJECTORS AND ACCEPTORS.

SECTION A. INTRODUCTION.

This Instruction deals with the basic principles of design of medium-wave impedance-matching networks, with particular reference to circuits used in aerial transformer houses (A.T.H.) and with rejectors and acceptors.

The purpose of the matching network in an aerial transformer house is to bring about an impedance match between the aerial and the feeder which is used to convey the R.F. power from the transmitter to the aerial. As explained in Section C, page 7, the correct termination for the feeder is a non-reactive resistance, equal in value to the characteristic impedance of the feeder and usually very different from the impedance of the aerial.

In practice, the setting-up of such networks is done usually with the aid of measurements taken on an R.F. bridge, very exact calculations not being attempted owing to the influence of factors the magnitude of which is not readily predictable, e.g. stray capacities between components and from components to earth, stray mutual couplings, etc. Calculations of networks, even though approximate, are, however, most necessary in order to reduce the amount of trial and error work in setting-up and to ensure that the components used are properly rated.

Facility in handling problems involving network calculations is very largely a matter of practice. Certain worked examples are contained in the text and a number of problems in Appendix B. As the worked examples in the text are intended to illustrate basic principles the calculations are carried out to a higher degree of accuracy than would, in many instances, be necessary for practical purposes (or even warranted in some situations that occur in field work).

As a general rule it can be taken that the accuracy obtainable by reasonably careful use of a 10-in. slide rule is adequate.

The Operator j.

Frequent use of j is made in this Instruction and the following information is given for the benefit of those not familiar with its significance and application.

j will first be considered as a vector operator. The basic principles of vector representation of alternating quantities will be found in standard text books (e.g. Admiralty Handbook of Wireless Telegraphy), to which reference should be made, if necessary.

Fig 1

By the convention of vector diagrams OV_1 and OV_2 of Fig. 1 could represent two voltages, equal in value but 180° out of phase.

Also, by convention

$OV_2 = -OV_1 = -1 \times OV_1$ ....(1)

Thus -1 used in this manner can be regarded as an " operator " producing a phase shift of 180° in a vector quantity.

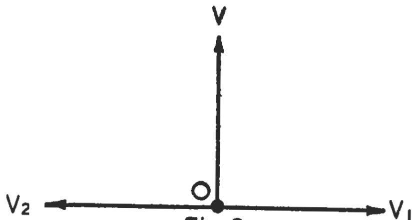


Fig 2

In Fig. 2 are shown the same two vectors OV_1 and OV_2, 180° out of phase with each other but, in addition, there is another vector OV, which represents a voltage of the same value as OV_1 but 90° out of phase with the latter.

What is the " operator " that would shift OV_1 90° to the position OV ?

Let j be the symbol for such an operator. Then,

$jOV_1 = OV$ .... (2)

Since multiplication by j produces a shift of 90° in an anti-clockwise direction, it follows that :—

$$jOV = OV_2$$

But, from equation (2), $OV = jOV_1$

$\therefore\ jOV = j(jOV_1) = j^2OV_1$

$\therefore\ OV_2 = j^2OV_1$

From equation (1)

$OV_2 = -1 \times OV_1$.

Whence,

$j^2 = -1$

$\therefore\ j = \sqrt{-1}$

Thus, following on from vector conventions, j is the 90° phase shift operator and, from the mathematical point of view, is equal to the square root of minus one.

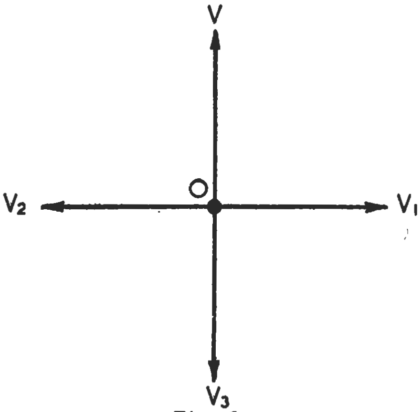


Fig 3

When considering 90° phase differences the matter of " lead " and " lag " arises and due regard must be paid to this in the application of operator j.

If in Fig. 3 $OV = jOV_1$, then $OV_3 = -jOV_1$.

This can be deduced as follows :—

$$OV_3 = jOV_2$$

But

$$OV_2 = -OV_1.$$

$$\therefore\ OV_3 = j(-OV_1) = -jOV_1.$$

Thus, if multiplication by +j produces a phase shift of 90° in an anti-clockwise sense, multiplication by −j will produce a phase shift of 90° in a clockwise sense.

j as an Algebraic Symbol.

The significance attached to j, as described above, does not in any way prevent the symbol being treated in equations and formulae in accordance with ordinary rules of algebra.

For example :—

$$(jA)\,(jB) = j^2AB$$

$$jA(-jB) = -j^2AB$$

$$\frac{jA}{jB} = \frac{A}{B}$$

j as $\sqrt{-1}$.

The square root of minus one is an imaginary quantity although the square of such a quantity is real.

From the fact that j is equal to $\sqrt{-1}$, it follows that :—

$$j^2 = -1$$

$$-j^2 = 1$$

$$(-j)^2 = -1$$

$$(-j)\,(+j) = 1$$

$$\frac{1}{j} = \frac{-j}{(j)\,(-j)} = -j$$

$$\frac{1}{-j} = \frac{j}{(-j)\,(j)} = +j$$

$$\frac{1}{j^2} = -1$$

$$\frac{1}{-j^2} = 1$$

The equalities given above are exceedingly useful for purposes of simplification of expressions involving j.

Complex Expressions.

A complex expression is one which contains both real and imaginary terms and is, in fact, of the general form A+jB.

A very useful fact to note is that if

$$A + jB = P + jQ$$

then

$$A = P$$

and

$$B = Q$$

Conjugate Expressions. Rationalisation.

A+jB and A−jB are said to be " conjugate " impedance expressions.

Note particularly the effect of multiplying together two conjugate expressions, thus :—

$$(A+jB)\,(A-jB) = A^2 - jAB + jAB + B^2$$
$$= A^2 + B^2$$

Advantage can be taken of the above to rationalise a complex denominator in a fractional expression.

Thus,

$$\frac{C}{A-jB} = \frac{C\,(A+jB)}{(A-jB)\,(A+jB)} = \frac{CA+jCB}{A^2+B^2}$$
$$= \frac{CA}{A^2+B^2} + j\,\frac{CB}{A^2+B^2}$$

Application of j to Impedance Expressions.

In Fig. 4(a) is shown an impedance consisting of a resistance R, in series with an inductance L (assumed to have negligible resistance). V is the voltage across the combination of R and L, I the current flowing through R and L, V_R the voltage

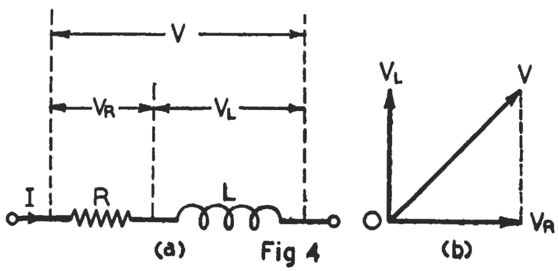


Fig 4

component across R and V_L the voltage component across L.

The voltages are represented in vector form in Fig. 4(b). From elementary considerations :—

$$V_R = RI \text{ volts.}$$

$$\text{and } V_L = \omega LI \text{ volts.}$$

V is not the arithmetical sum of V_R and V_L owing to the 90° phase difference between these two voltage components. V is, in fact, the " vectorial sum " of V_R and V_L (90° out of phase with V_R) which can be expressed in the following manner ;—

$$V = V_R + jV_L$$

Calling the impedance of R and L in series Z, it follows that $ZI = RI + j\omega LI$.

Eliminating I gives :—

$$Z = R + j\omega L$$

In a similar manner it can be shown that if Z is made up of a resistance in series with a capacity,

$$Z = R - j\frac{1}{\omega C}$$

Impedance in General Terms.

In the text of this Instruction R, Z and X are used as general symbols for resistance, impedance and reactance respectively, appropriate suffix letters being added when it is required to give any particular signficance to the symbols, e.g. R_s and R_p, representing series and parallel connected resistances respectively.

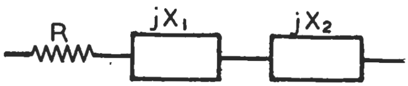


Fig 5

Fig. 5 schematically represents an impedance combination consisting of a resistance R in series with two reactances X_1 and X_2 where either X_1 or X_2 may be a reactance due to inductance or to capacity.

In general terms,

$$Z = R + j(X_1 + X_2)$$

In applying the above expression to any specific case, due regard must be given to the signs of the reactances, using a positive sign with any reactance value due to inductance and a negative sign with any reactance value due to capacity.

Thus, with reference to Fig. 5, if R is a resistance of 200 ohms, X_1 a reactance of 300 ohms due to an inductance and X_2 a reactance of 450 ohms due to a condenser,

$$Z = 200 + j\,(300 - 450)$$
$$= 200 - j\ 150 \text{ ohms}$$

Impedance expressions stated in general terms appear frequently in the text of this Instruction and *attention to signs must be given whenever such expressions are applied in calculations relating to specific cases.*

Series and Parallel Impedance Combinations

In order to avoid possible confusion between series and parallel impedance expressions, when given in complex form, the sign // is used to distinguish the parallel cases.

Thus 24+j200 ohms refers to an impedance combination of 24 ohms resistance in series with 200 ohms inductive reactance. 24//j200 ohms refers to an impedance combination comprising 24 ohms resistance in parallel with 200 ohms inductive reactance.

SECTION B. AERIAL DRIVING POINT IMPEDANCE.

The design of an A.T.H. network is based primarily upon :—

(*a*) The impedance of the aerial (looking into it at the driving point) at the carrier frequency.

(*b*) The characteristic impedance of the feeder.

In practice the aerial impedance is determined by bridge measurement. Under ideal conditions, when the dimensions of the aerial and the operating frequency alone control the impedance, the latter might be calculated to a satisfactory degree of accuracy, but in practice the aerial impedance is influenced by masts, stays, other aerials, etc., on the site, and measurement is always necessary. The influence that neighbouring conductors have upon the impedance of an aerial depends upon a number of factors in addition to relative distances and orientations, e.g. resonant frequencies, R.F. resistances, etc. When a bridge measurement of the driving-point impedance of an aerial is being made care should be taken to ensure that all other aerials on the site are terminated correctly and that metal masts are in their normal condition as regards earthing or insulation.

Over the medium-wave frequency band the driving-point impedance of an aerial goes through considerable changes as regards both the resistive and the reactive components. The reactance will be zero at frequencies corresponding with quarter-wave and half-wave operation. At frequencies lower than that corresponding with quarter-wave operation the reactance will be negative, at frequencies between the quarter-wave

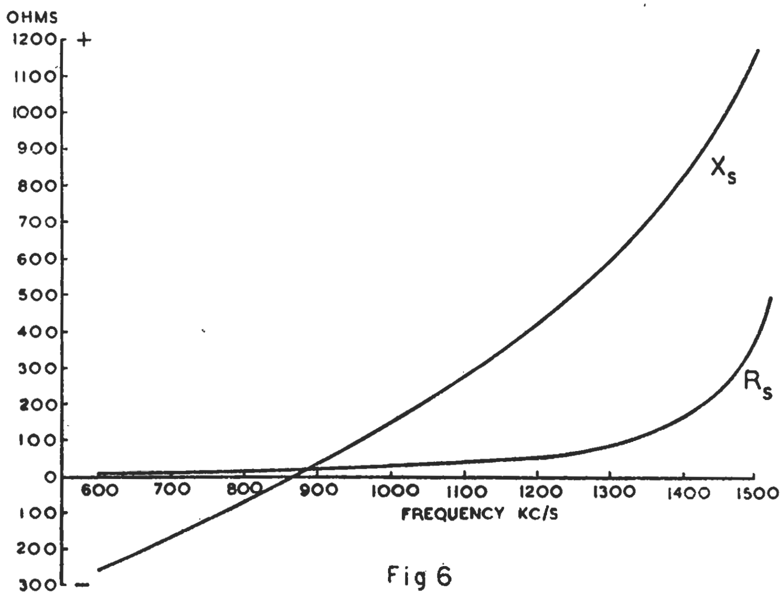


Fig 6

and half-wave conditions it will be positive, becoming negative again between half-wave and three-quarter wave.

The resistive component of the aerial impedance is a value which includes the radiation resistance and the loss resistances. As far as the driving-point impedance of the aerial is concerned, it should be noted that the resistive component is low under quarter-wave conditions and high under half-wave conditions.

When bridge measurements are taken of the driving-point impedances of aerials on BBC sites, it is usual to make a frequency run and to plot the impedance characteristic over the waveband. Figs. 6 and 7 are two examples.

Fig. 6 is plotted from the results of bridge measurements of Z_s taken on a certain small sloping single-wire aerial. The length of the top portion is 150 ft. and the average height 75 ft. A single-wire downlead, 150 ft. in length, is attached to the centre of the top portion. It will be seen from the figure that quarter-wave conditions occur at a frequency of 870 kc/s, the reactance being zero and the resistance low at this particular frequency.

The curves of Fig. 7 show the impedance char-

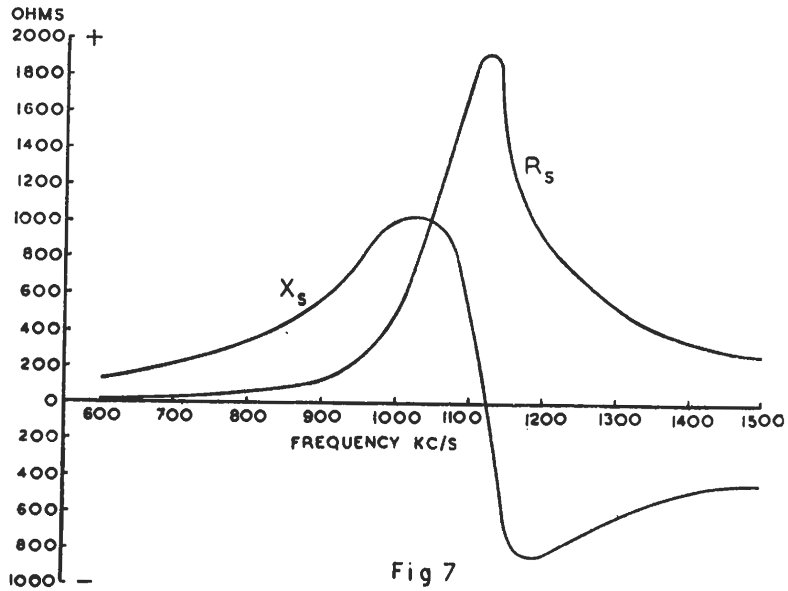


Fig 7

acteristics of a certain large aerial of T type. The top span is of 4-wire construction with a length of 417 ft., while the downlead consists of a vertical 9-wire cage 160 ft. in length. It will be seen from the figure that half-wave conditions occur at a frequency of 1,125 kc/s, the reactance being zero and the resistance high at this frequency.

Figs. 6 and 7 are reproduced for illustrative purposes and mainly with the object of giving emphasis to the fact that the driving-point impedance of any aerial varies very considerably over the medium-wave band.

SECTION C. R.F. FEEDERS.

In the common circumstance where the aerial is situated at such a distance from the transmitter building that the aerial downlead cannot be brought direct to the transmitter, the R.F. power is conveyed from the transmitter to the aerial by a feeder line.

An R.F. feeder must necessarily introduce a certain amount of attenuation due to resistance and insulation leakage, and to radiation. In the following description, however, the assumption will be made that the feeder is dissipationless. This will not be strictly correct in practice, but where feeders used by BBC stations are concerned, usually the losses on medium and long waves due to causes mentioned above are negligible and will, therefore, not be considered in this Instruction.

Characteristic Impedance.

Suppose a source of R.F. power is connected to one end of a long feeder, the other end of which is open. Since the feeder is assumed to be dissipationless and there is no terminating load, there will be nothing to absorb the power. Under these conditions power arriving at the open end will be reflected back along the feeder, returning to the source. At any point between the source and the open end of the feeder there will thus be waves moving forward from the source and also waves moving back towards the source. These forward and backward waves will interact in such a manner that *standing waves* are produced on the feeder, i.e. stationary current and voltage nodal (minimum) and antinodal (maximum) points will be set up along the feeder. The voltage nodal points will be spaced at half-wavelength distances and the antinodal points will also be spaced at half-wavelength distances, while there will be a quarter-wavelength distance between any node and an adjacent antinode. As there is no power absorption there will be 90° phase difference between the current and voltage fluctuations at any point on the feeder. Thus a current node will coincide with a voltage antinode and a voltage node will coincide with a current antinode.

In the case under consideration there will be a current node and a voltage antinode at the open end of the feeder. Suppose that, instead of the feeder being open ended it is short circuited. Again, there can be no power absorption and standing waves will be produced on the feeder. In this case the short-circuited end of the feeder will be a point at which there will be a current antinode and a voltage node.

If a resistance load is connected across the end of the feeder remote from the source, there will be absorption of power in this load, but it will be found that the resistance value of the load must be of a certain critical value if there is to be no reflection. If the load resistance is greater or smaller than this critical value, there will be part absorption and part reflection of power, the reflected component setting up standing waves on the feeder.

The value of terminating resistance, which will give complete absorption of power and no standing waves on the feeder, is equal to the *characteristic impedance* of the feeder.

The characteristic impedance of a dissipationless feeder is a non-reactive resistance value of :

$$\sqrt{\frac{L}{C}} \text{ ohms}$$

where L = inductance per unit length of feeder

C = capacity ,, ,, ,, ,, ,,

The derivation of this formula will be found in a number of standard text books.

It will be seen therefore that the characteristic impedance of a dissipationless feeder will be dependent upon the cross-section and spacing of the conductors and upon the conductor configuration, but it will *not* depend upon the length of the feeder.

Formulae are given in Appendix C, page 55, which enable approximate calculation to be made of the characteristic impedances of certain types of R.F. feeder. Such differences as are found between calculated and measured values are due to factors, dependent upon the particular physical construction employed, such as effects of supporting insulators, capacity to earth (in the case of open wire feeders), etc.

Measurement of Characteristic Impedance

The characteristic impedance of a feeder line can be measured as follows :—

A convenient length of feeder should first be short-circuited at one end, while a measurement of the impedance (series aspect) looking into the feeder, is made by means of an R.F. bridge connected to the other end. Next a measurement should be made with the end of the feeder remote from the bridge open-circuited.

Then :—

$$Z_o = \sqrt{Z_{sc} Z_{oc}}$$

where Z_o = characteristic impedance of feeder

Z_{sc} = bridge measurement with far end of feeder short-circuited.

Z_{oc} = bridge measurement with far end of feeder open-circuited.

Note: The symbol R_o will be used for characteristic impedance in diagrams and examples in this Instruction because in all cases the characteristic impedance of a feeder will be assumed to be purely resistive.

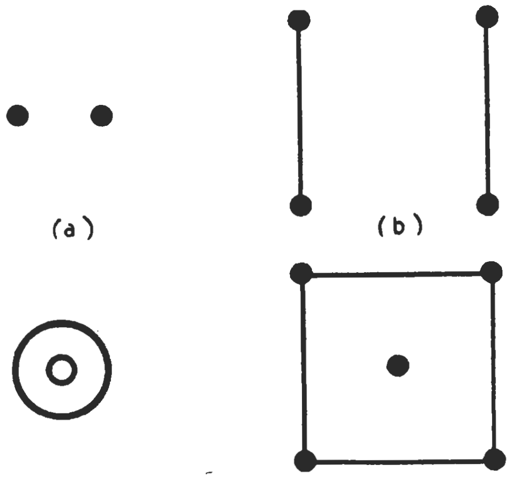


Fig 8

Types of R.F. Feeder.

The various types of feeders may be broadly classified into two groups: (i) Balanced with respect to earth, (ii) Unbalanced with respect to earth.

Fig. 8 shows cross-sectional sketches of certain feeders in common use at BBC stations.

Sketch (a) illustrates the 2-wire balanced feeder. Using 19/.064 wire with a spacing of 18 in. between the two conductors, the measured characteristic impedance of a certain feeder of this type is 550 ohms. The calculated value, using formula (30c), Appendix C, is 566 ohms.

Sketch (b) illustrates a 4-wire feeder with equal horizontal and vertical spacing and with the wires parallelled in two vertical pairs. A certain feeder of this type constructed of No. 6 S.W.G. wire with 12 in. spacing has a measured characteristic impedance of 320 ohms. The calculated value, using formula (31C), Appendix C, is 310 ohms.

Sketch (c) illustrates the concentric feeder. This feeder is of unbalanced type, the outer conductor being earthed. At the pre-war Regional/National transmitting stations concentric feeders were installed, employing an inner copper tubular conductor of 1.375 in. outside diameter and an outer tubular conductor of 5 in. inside diameter. The measured characteristic impedance is 80 ohms. The calculated value, using formula (32C), Appendix C, is 77 ohms.

Sketch (d) illustrates the 5-wire unbalanced feeder. With this type of feeder the four outer wires are parallelled and earthed. A certain feeder of this type employing four outer conductors of No. 6 S.W.G. wire spaced 12 in. horizontally and vertically and with a central inner conductor of 19/.044 wire has a measured characteristic impedance of 300 ohms. Using formula (33C), Appendix C, and assuming all conductors of the diameter of 19/.044 wire, gives a calculated value of 310 ohms. The fact that in this particular case there is a slight difference between the diameters of the inner and the outer conductors can be disregarded in view of the fact that the calculation by formula represents an approximation only.

Matching.

In view of the information given in Sections B and C, it is now possible to amplify the information given in Section A regarding the matching network in an aerial transformer house.

The two impedances which have to be matched by the network are the aerial impedance on the one hand and the characteristic impedance of the feeder on the other. This implies that the combination of aerial and network must in the ideal case present to the feeder a load of Z_o ohms, where Z_o is the characteristic impedance of the feeder.

Except when the aerial is being operated under quarter-wave or half-wave conditions the aerial impedance will be complex. (The term complex is used here in its mathematical sense and does not mean " complicated.") The characteristic impedance of the feeder, however, will be non-reactive (assuming a dissipationless feeder). It will be seen, therefore, that one possibility regarding an A.T.H. network is that it consists of (a) reactive elements, the purpose of which is to balance out the reactive component of the aerial impedance and (b) reactive elements, the purpose of which is to convert the resulting non-reactive resistance to a value equal to the characteristic impedance of the feeder. The network used to match the impedance of the aerial to the characteristic impedance of the feeder is often called a *transducer*.

A complete A.T.H. network, however, cannot always be regarded in such a simple manner as described above, i.e. regarded as made up of two separate parts, one balancing out aerial reactance and the other providing the necessary resistance transformation, although this sometimes applies. In some cases, for example, the whole or part of the aerial reactance is incorporated in the transducer and when this is possible the theoretical considerations are a little more complicated, although the practical set-up may be greatly simplified and costs reduced. This matter is considered in detail in Section H (A.T.H. circuits). As a preliminary, however, the theory of transducers will be dealt with without reference to aerial reactance.

SECTION D. SERIES AND PARALLEL IMPEDANCE EQUIVALENTS.

Any impedance represented by a resistance in series with a reactance is equivalent, at any given single frequency, to a particular parallel combination of resistance and reactance. The converse also applies.

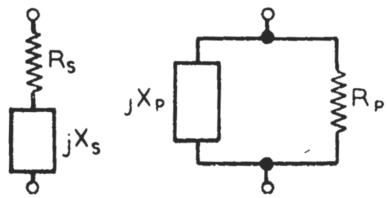


Fig 9

In Fig. 9 (a) is shown a resistance, R_s, in series with a reactance, X_s, the latter being shown in block schematic form, since the argument applies equally well whether the reactance be positive or negative, provided that the reactance sign is the same in both the series and the parallel cases. In Fig. 9 (b) is shown a resistance, R_p, in parallel with a reactance, X_p. The two impedance combinations of Figs. 9 (a) and 9 (b), respectively, can be made equivalent to each other, at any given single frequency, provided that certain relations of values are established, the correct relations being derived later in this section.

Assuming that the correct relation of values exists, it follows from the statements made above that the series combination of Fig. 9 (a) could be replaced by the parallel combination of Fig. 9 (b) and vice versa. In point of fact, if two series and parallel equivalent impedances were individually made up and separately boxed without distinguishing marks, it would be impossible by an R.F. bridge measurement of impedance (made at the frequency of equivalence) to determine which box contained the series combination and which the parallel combination.

It must be emphasized that the equivalence holds good only at the one particular frequency.

A far-reaching consequence of the series-parallel equivalent impedance rule is that, even though an impedance may actually consist of elements connected in series, it is perfectly legitimate to treat it as though it existed in the form of its parallel equivalent, and the converse holds good.

Series to Parallel Equivalent Impedance Conversion.

The parallel combination of Fig. 9 (b) will be equivalent to the series combination of Fig. 9 (a) if,

$$R_p = \frac{R_s^2 + X_s^2}{R_s} \qquad \text{.......(3)}$$

and

$$X_p = \frac{R_s^2 + X_s^2}{X_s} \qquad \text{.......(4)}$$

the sign of X_p being the same as the sign of X_s.

For the purpose of slide-rule calculation the expressions for R_p and X_p are more convenient if given in the following form :—

$$R_p = R_s\left(1 + \frac{X_s^2}{R_s^2}\right) \qquad \text{.......(5)}$$

and

$$X_p = X_s\left(1 + \frac{R_s^2}{X_s^2}\right) \qquad \text{.......(6)}$$

The derivation of the above formulae is given in Appendix A, page 47.

Example 1: What is the parallel equivalent of 38−j76 ohms?

Answer:

From equation (3),

$$R_p=38\left(1+\frac{(-76)^2}{38^2}\right)=38\times5=190.$$

From equation (4),

$$X_p=-76\left(1+\frac{38^2}{(-76)^2}\right)=-76\times1.25=-95$$

∴ Parallel equivalent of 38−j76 ohms is 190//−j95 ohms.

A fact that can be applied very usefully for numerical checking purposes is that the product of the series and parallel equivalent resistance values is equal to the product of the series and parallel equivalent reactance values.

i.e. $R_pR_s=X_pX_s$

That this is true can be seen by inspection of equations (3) and (4).

From equation (3), $R_pR_s=R_s^2+X_s^2$

From equation (4), $X_pX_s=R_s^2+X_s^2$

$$\therefore\ R_pR_s=X_pX_s \qquad \text{.................................(7)}$$

In example 1, $R_pR_s=190\times38=7220$, while $X_pX_s=-95\times-76=7220$.

Alternative Values of X_s for Given Value of X_p, R_s being Constant.

A case of some practical significance is that in which an impedance R_s+jX_s has a constant value of R_s, but X_s is variable and of either positive or negative sign. It can be shown that in these circumstances X_s may in general be given either of two alternative values which will give rise to the same value of equivalent parallel reactance X_p, which will always be of the same sign as X_s. The resulting values of equivalent parallel resistance R_p will, of course, be different in the two cases. These two values of X_s are given by :—

$$X_{s_1}=\frac{X_p}{2}+\sqrt{\left(\frac{X_p}{2}+R_s\right)\left(\frac{X_p}{2}-R_s\right)} \qquad \text{......(8)}$$

and

$$X_{s_2}=\frac{X_p}{2}-\sqrt{\left(\frac{X_p}{2}+R_s\right)\left(\frac{X_p}{2}-R_s\right)} \qquad \text{......(9)}$$

If, however, X_p is numerically equal to $2R_s$, the two values of X_s become identical and numerically equal to R_s. If X_p is numerically less than $2R_s$, no real value of X_s can be found to satisfy the required condition, because the quantity under the square root sign becomes negative.

The derivation of the above formulae is given in Appendix A, page 47.

Example 2: If $Z_s=124+jX_s$ ohms, what values of X_s will give an equivalent parallel reactance of 520 ohms?

Answer:

From equations (8) and (9)

$$X_s=260\pm\sqrt{384\times136}$$
$$=260\pm\sqrt{52{,}220}$$
$$=260\pm228.5$$
$$\therefore\ X_s=488.5 \text{ or } 31.5 \text{ ohms.}$$

Of the two values of X_s, which give the same value of X_p (R_s being constant), one is greater than R_s, while the other is smaller. Further, if the larger value is n times R_s then the smaller value will be 1/nth of R_s.

Example 2 illustrates this fact for 488.5 is equal to 124×3.94, while 31.5 is equal to 124/3.94.

That both nR_s and R_s/n are values of X_s giving the same value of X_p can be proved by substituting first $X_s=nR_s$ and then $X_s=R_s/n$ in equation (4). Substitution in equation (3) gives the two resulting values of R_p.

Parallel to Series Equivalent Impedance Conversion.

The series combination of Fig. 9 (a) will be equivalent to the parallel combination of Fig. 9 (b) if

$$R_s=\frac{R_pX_p^2}{R_p^2+X_p^2} \qquad \text{.................................(10)}$$

and

$$X_s=\frac{R_p^2X_p}{R_p^2+X_p^2} \qquad \text{.................................(11}$$

the sign of X_s being the same as the sign of X_p.

For the purposes of slide-rule calculation the expressions for R_s and X_s are more convenient if given in the following form :—

$$R_s=\frac{R_p}{1+R_p^2/X_p^2} \qquad \text{.................................(12)}$$

and

$$X_s = \frac{X_p}{1+X_p^2/R_p^2} \quad \ldots\ldots\ldots\ldots(13)$$

The derivation of the above formulae is given in Appendix A, page 47.

Example 3: What is the series equivalent of 240//j85 ohms?

Answer:

From equation (12),

$$R_s = \frac{240}{1+240^2/85^2} = \frac{240}{1+7.98}$$
$$= \frac{240}{8.98} = 26.7 \text{ ohms}$$

From equation (13),

$$X_s = \frac{85}{1+85^2/240^2} = \frac{85}{1+0.126}$$
$$= \frac{85}{1.126} = 75.5 \text{ ohms}$$

∴ Series equivalent of 240//j85 ohms is 26.7+j75.5 ohms.

It has already been shown (page 10), with the aid of equations (3) and (4), that $R_pR_s = X_pX_s$. Inspection of equations (10) and (11) again shows that $R_pR_s = X_pX_s$. It is to be noted, however, that equations (10) and (11) do not provide a proof of the statement independently of equations (3) and (4), since the former pair of equations are derived from the same original equation as are the latter.

Alternative Values of X_p for Given Value of X_s, R_p being Constant.

In the case where an impedance $R_p//jX_p$ has a constant value of R_p, but X_p is variable and of either positive or negative sign, it can be shown that in general the same value of equivalent series reactance, X_s, is produced when X_p is given either of two alternative values. The resulting values of equivalent series resistance R_s will, of course, be different in the two cases. These two values of X_p are given by :—

$$X_{p1} = \frac{R_p}{2X_s}\left[R_p + \sqrt{(R_p+2X_s)\,(R_p-2X_s)}\right] \ldots(14)$$

$$X_{p2} = \frac{R_p}{2X_s}\left[R_p - \sqrt{(R_p+2X_s)\,(R_p-2X_s)}\right] \ldots(15)$$

If, however, X_s is numerically equal to $\frac{1}{2}R_p$, the two values of X_p become identical and numerically equal to R_p. If X_s is numerically greater than $\frac{1}{2}R_p$, no real value of X_p can be found to satisfy the required condition because the quantity under the square root sign becomes negative.

The derivation of the above formulae is given in Appendix A, page 47.

Example 4: If $Z_p = 124//jX_p$ ohms, what values of X_p will give an equivalent series reactance of −50 ohms?

Answer:

From equations (14) and (15),

$$X_p = -\frac{124}{100}\left[124 \pm \sqrt{224 \times 24}\right]$$
$$= -1.24\,[124 \pm 73.31]$$

$\therefore X_p = -224.6$ or -62.87 ohms.

Example 5: What values of series equivalent resistance are given by the two values of X_p in Example 4?

Answer:

From equation (7),

$$R_s = \frac{X_pX_s}{R_p}$$
$$= \frac{244.6 \times 50}{124} \text{ or } \frac{62.87 \times 50}{124}$$

$\therefore R_s = 98.65$ or 25.36 ohms.

The two values of X_p which give the same value of series equivalent reactance are related to R_p in the following manner :—

If one of the X_p values is nR_p, the other will be R_p/n. Proof can be obtained by substituting, first $X_p = nR_p$ and then $X_p = R_p/n$ in equation (11).

SECTION E. L-TYPE TRANSFORMING NETWORKS.

Transforming Downwards.

The first case to be considered will be that where it is required to transform a resistance R_A to a *smaller* resistance value R_B. The upwards transforming case will be dealt with afterwards.

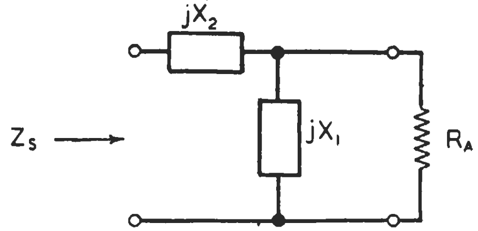


Fig 10

Suppose a reactance X_1 is connected in shunt with R_A, as in Fig. 10, and let the series equivalent impedance of R_A and X_1 in parallel be R_s+jX_s.

R_s must necessarily be smaller than R_A and, by suitable choice of X_1, can be made any value between R_A and zero. Let the value of X_1 be chosen so that $R_s=R_B$. The series equivalent impedance of R_A and X_1 in parallel is then R_B+jX_s.

If a reactance X_2 is connected in series with the parallel combination of R_A and X_1, then the series equivalent impedance looking into the terminals of the network is,

$$Z_s=R_B+j(X_s+X_2)$$

If X_2 is made equal in value and opposite in sign to X_s, that is $X_2=-X_s$, then,

$$Z_s=R_B$$

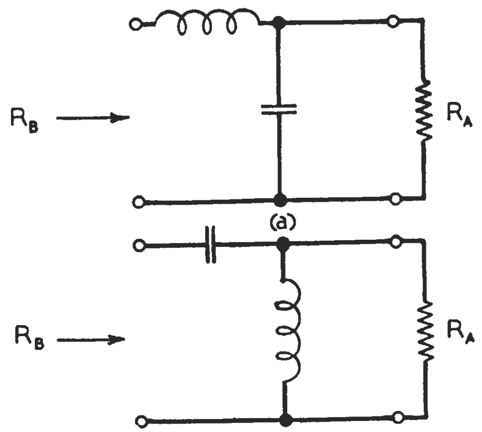
which is the desired condition.

The L network composed of the reactances X_1 and X_2 thus transforms R_A down to R_B.

It will be seen that a downwards L transducer consists of (i) a shunt reactive element, the value of which is such as to make the equivalent series resistance equal to the required value of transformed resistance, and (ii) a series reactive element of opposite sign to the shunt element and equal in value to the equivalent series reactance.

Since the shunt and series elements must be of opposite sign, there are two alternative basic forms of the downwards L transducer, as shown in Figs. 11 (a) and 11 (b).

Where A.T.H. circuits are concerned, preference is generally given to the network of Fig. 11 (a). To have capacity in the shunt arm and inductance in the series arm gives a network providing attenuation of harmonic frequencies of the carrier.



(b)
Fig 11

This is important if the output circuits of the transmitter do not themselves reduce the harmonic content of the carrier output to negligible proportions. Apart from this consideration, which will be of greater or less importance according to circumstances, the network of Fig. 11 (a) avoids the use of a series condenser.

The installation of a series condenser inevitably introduces capacity to earth. High power condensers have large cases often connected to one side of the condenser—and it is difficult to prevent the capacity to earth being sufficiently large to modify appreciably the performance of the network and in some cases it may prove awkward to obtain the required final result. In addition, large capacity to earth may give heavy losses in surrounding brickwork, etc., unless screening is employed.

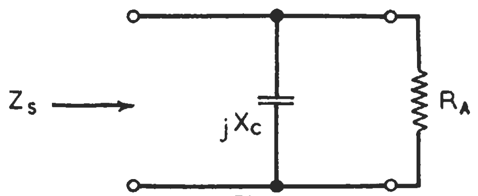


Fig 12

Dealing more specifically with downwards L transducers of the form shown in Fig. 11 (a), consider a case in which the shunt condenser is connected but the series inductance is omitted, as in Fig. 12.

From equations (10) and (11), page 10,

$$Z_s=\frac{R_A X_c^2}{R_A^2+X_c^2}+j\frac{R_A^2 X_c}{R_A^2+X_c^2} \quad \text{............(16)}$$

Let the capacity be so chosen that,

$$\frac{R_A X_C^2}{R_A^2+X_C^2}=R_B$$

then,

$$Z_s=R_B+j\frac{R_A^2 X_C}{R_A^2+X_C^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots(17)$$

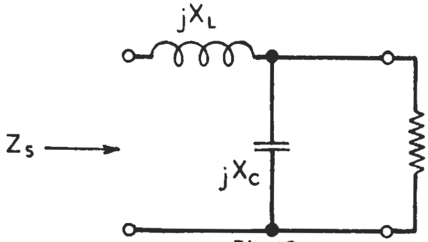


Fig 13

If a series inductance is now connected as shown in Fig. 13,

$$Z_s=R_B+j\left(X_L+\frac{R_A^2 X_C}{R_A^2+X_C^2}\right) \ldots\ldots\ldots\ldots(18)$$

If the inductance is so chosen that,

$$X_L=-\frac{R_A^2 X_C}{R_A^2+X_C^2}$$

then,

$$Z_s=R_B$$

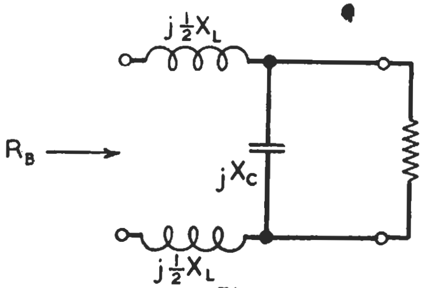


Fig 14

The circuit of Fig. 13 is of unbalanced type. Should it be required that the circuit shall be balanced the total inductance can be split into two halves, connected as shown in Fig. 14.

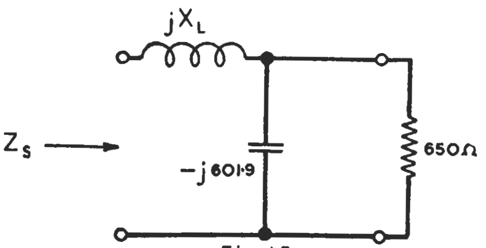


Fig 15

Example 6: What are the values of Z_s in Fig. 15 when X_L has the following values :—

(*a*) +210 ohms
(*b*) +400 ohms
(*c*) +324.1 ohms

Answer:

From equation (12) the equivalent series resistance of 650//−j601.9 ohms is,

$$\frac{650}{1+650^2/(-601.9)^2}=\frac{650}{2.166}=300 \text{ ohms}$$

From equation (13) the equivalent series reactance of 650//−j601.9 ohms is,

$$\frac{-601.9}{1+(-601.9)^2/650^2}=\frac{-601.9}{1.857}=-324.1 \text{ ohms}$$

Thus, in case (*a*) $Z_s=300-j324.1+j210$
$=300-j114.1$ ohms

In case (*b*) $Z_s=300-j324.1+j400$
$=300+j75.9$ ohms

In case (*c*) $Z_s=300-j324.1+j324.1$
$=300+j0$ ohms

Case (*c*) of example 6 is a case where there is a transformation from 650 ohms resistance to 300 ohms resistance. In case (*a*) the inductance is not large enough to balance out the equivalent series reactance of the condenser and 650 ohms resistance in parallel, while in case (*b*) it is too large and Z_s has consequently a component of positive reactance.

It will be noted that the assumption is made in example 6 that the equivalent series resistance, 300 ohms, does not change with variation of the inductance. This would only be the case if the inductance had no R.F. resistance and if stray capacities between components and from components to earth did not exist. However, with a low-loss coil carefully mounted to minimise strays the change of R_s will be very slight. As approximate calculations only are dealt with in this Instruction, the results given in example 6 are permissible.

Design Formulae. Downwards L Transducer.

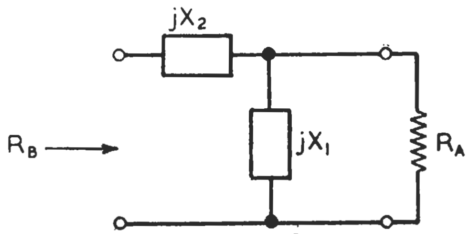


Fig 16

Fig. 16 shows an L-type transducer which is transforming the resistance value R_A down to a lower value R_B. The required values for X_1 and

X_2 can be calculated by the following simple formulae :—

$$X_1 = \pm\frac{mR_B}{\sqrt{m-1}} \quad \text{.............. (19)}$$

$$X_2 = \mp R_B\sqrt{m-1} \quad \text{.............. (20)}$$

where $m = \frac{R_A}{R_B}$

The derivation of the above formulae is given in Appendix A, page 48.

Example 7 : What are the reactance values for an L-type transducer of the form illustrated in Fig. 11 (a), which will transform a resistance of 550 ohms to a resistance of 80 ohms ?

Answer :

$$m = \frac{550}{80} = 6.875$$

$$\sqrt{m-1} = 2.424$$

From equation (20),

$$X_L = +80 \times 2.424$$
$$= +193.9 \text{ ohms}$$

From equation (19),

$$X_C = -\frac{6.875 \times 80}{2.424}$$
$$= -227 \text{ ohms}$$

Example 8 : Calculate the L and C values and draw the circuit of a transducer of the form illustrated in Fig. 11 (a), which will transform a resistance of 1900 ohms to a resistance of 550 ohms at a frequency of 668 kc/s.

Answer :

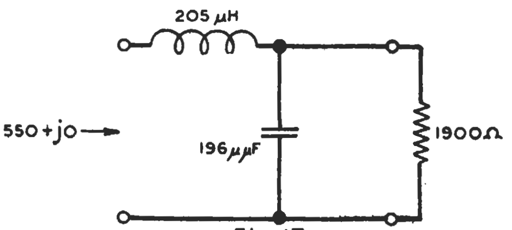


Fig 17

$$m = \frac{1900}{550} = 3.45$$

$$\sqrt{m-1} = 1.565$$

From equation (20),

$$X_L = +550 \times 1.565$$
$$= +860.8 \text{ ohms}$$

$$\therefore L = \frac{860.8 \times 10^3}{2\pi \times 668} = 205\mu H$$

From equation (19),

$$X_C = -\frac{3.45 \times 550}{1.565}$$
$$= -1213 \text{ ohms}$$

$$\therefore C = \frac{10^9}{2\pi \times 668 \times 1213} = 196\mu\mu F$$

The diagram is given in Fig. 17.

Example 9 : If a 300 μμF condenser were substituted for the condenser shown in Fig. 17, what change of inductance would be required to restore unity power factor (looking into the terminals of the network), and what change would occur in the transformed resistance value ? Assume that the frequency remains at 668 kc/s.

Answer :

The reactance of 300 μμF at 668 kc/s is,

$$X_C = -\frac{10^9}{2\pi \times 668 \times 300} = -793.4 \text{ ohms}$$

From equation (13) the equivalent series reactance of 1900// −j793.4 ohms is,

$$X_s = \frac{-793.4}{1 + (-793.4)^2/1900^2} = -675.7 \text{ ohms}$$

For unity power factor at the input terminals of the network it will be necessary to adjust the inductance to provide a reactance of +675.7 ohms.

At 668 kc/s this will require an inductance value of,

$$L = \frac{675.7 \times 10^3}{2\pi \times 668} = 160.7 \ \mu H$$

Thus the inductance must be reduced from 205 μH to 160.7 μH.

From equation (12), the equivalent series resistance of 1900// −j793.4 ohms is,

$$R_s = \frac{1900}{1 + 1900^2/(-793.4)^2} = 282 \text{ ohms}$$

Thus the value of transformed resistance would drop from 550 ohms to 282 ohms.

The Practical Setting-up of a Downwards L Transducer.

The performance of a transducer may be found in practice to differ somewhat from that assumed in the " paper design " for the reason that the design formulae given in this Instruction take no account of stray capacities, inductances and couplings, nor of R.F. resistance and dielectric losses.

The effects of "strays" can be considerable and it is necessary to construct the network with care, adopting as far as circumstances will permit all the usual methods, familiar to transmitter engineers, of keeping "strays" at a minimum. In particular, capacities from coils to earth should be kept down as low as possible.

Fixed condensers are frequently used as the shunt elements in L type networks. The nearest capacity value available may in some cases differ from the required value sufficiently to prevent the desired impedance match being obtained without modification of the network. A method of overcoming this difficulty is as follows:—

If the condenser introduces too much negative reactance (i.e. if the capacity is too small), an inductance should be connected in series with it and adjusted to a value which cancels the excess negative reactance. In a normal case only a small value of inductance will be required.

If the reactance of the condenser is too small (i.e. if the capacity is too large), the required value of negative reactance can be secured by the use of a suitable value of inductance connected in shunt with the condenser. In a normal case the inductance value required will be comparatively large and this artifice is consequently not so convenient as that described for the smaller capacity case.

The adjustments that are required in setting up a terminated transducer to give the required value of transformed resistance are best made with the aid of an R.F. bridge.

It should be noted that the standard BBC bridge is one that provides impedance measurements in ***parallel terms***. Thus, if the bridge is used to measure an impedance combination made up of resistance and reactance connected in series the direct results of the bridge measurement will give the equivalent parallel resistance and reactance of the series combination. In cases where the series values are required, these can be obtained from the measured equivalent parallel values by simple calculation, as described in Section D of this Instruction.

The practical adjusting of a downwards L transducer, terminated by a resistance R_A, to give the required value of transformed resistance will be considered in relation to two possible cases:—

Case 1.—Network of the form shown in Fig. 11 (a). Shunt capacity fixed. Series inductance variable.

Case 2.—Network of the form shown in Fig. 11 (a). Shunt capacity variable. Series inductance variable.

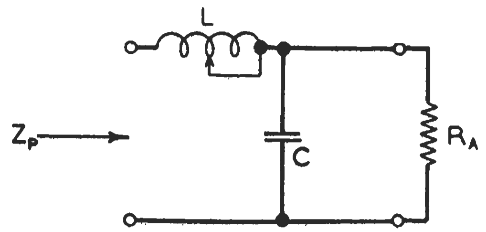


Fig 18

Case 1.—Shunt capacity fixed. Series inductance variable. Fig. 18.

With the bridge connected to measure Z_P, Fig. 18, adjust L until unity power factor is obtained at the input terminals of the network. That is, adjust L until the bridge indicates that the reactive component of Z_P is infinite. In this connection it must be remembered that $R//j\infty$ is equivalent to $R+j0$.

The resistance value as measured by the bridge is the value of the transformed resistance.

If it is found that there is a greater difference than is permissible between the expected and measured values of transformed resistance a check should be made upon the equivalent series resistance component of $R_A//jX_C$. To do this, disconnect L from C and connect the bridge directly across C. From the bridge measurements calculate the equivalent series resistance value.

If L has negligible influence upon the transformed resistance value the series resistance, determined as above, will be equal to the transformed resistance as previously measured. If this proves to be the case then an incorrect value of C should be presumed. If, however, the test shows that L is responsible for the incorrect value of transformed resistance then the existence of excessive "strays" associated with L should be suspected.

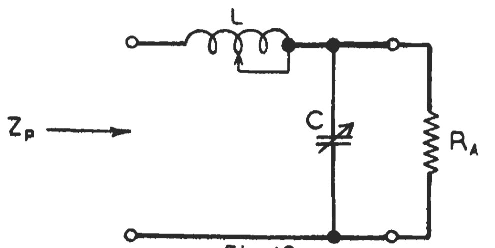


Fig 19

Case 2.—Shunt capacity variable. Series inductance variable. Fig. 19.

Assuming that "strays" have been minimised by careful construction of the circuit, the correct adjustments of C and L can be made as follows :—

Adjust the condenser to a setting that is considered to be correct. With the bridge connected to measure Z_P, Fig. 19, adjust L for unity power factor at the input terminals of the network.

If the condenser setting approximates to the correct value, the measured value of transformed resistance should now approximate to the required value. Fine adjustment of the transformed resistance value can then be made by fine adjustment of C, re-adjusting L for unity power factor with each change of C.

Transforming Upwards.

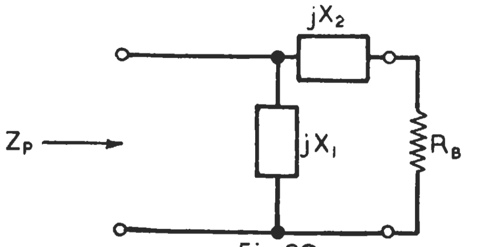


Fig 20

Consideration will now be given to the case where it is required to transform a resistance R_B to a *greater* resistance value, R_A.

Let a reactance X_2 be connected in series with R_B as in Fig. 20, and let the equivalent parallel impedance of R_B and X_2 in series be $R_P//jX_P$. R_P will be greater than R_B and can be made equal to R_A by suitable choice of X_2.

If a shunt reactance, X_1, is connected as shown in Fig. 20, the equivalent parallel impedance, looking into the terminals of the network, is

$$Z_P = R_A//j\left(\frac{X_1\ X_P}{X_1 + X_P}\right) \quad \text{......................(21)}$$

If X_1 is a reactance of opposite sign to X_2 (and therefore to X_P) and is so adjusted that $X_1 = -X_P$, then the expression in brackets becomes infinity. Thus Z_P is equivalent to a resistance R_A shunted by infinite reactance. In other words,

$$Z_P = R_A$$

which is the desired condition.

Thus, an upwards L transducer consists of a series reactive element, the value of which is such as to make the equivalent parallel resistance equal to the required value of transformed resistance, and (ii) a shunt reactive element of opposite sign to the series element and equal in value to the equivalent parallel reactance.

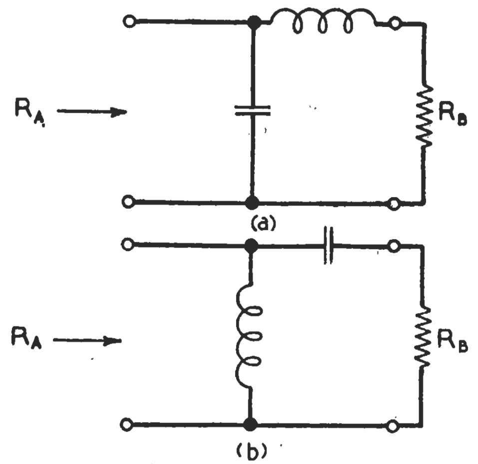


Fig 21

The two basic forms of the upwards L transducer are shown in Figs. 21 (a) and 21 (b). The more commonly used circuit is that of Fig. 21 (a), but in an A.T.H. network that is not required to contribute to the attenuation of harmonics the circuit of Fig. 21 (b) may be used.

It does not necessarily follow where A.T.H. circuits are concerned that a basic schematic of the type of Fig. 21 (b) necessarily implies the use of a series-connected condenser. In some cases the reactive component of the aerial driving-point impedance may provide the required negative reactance without a condenser being necessary.

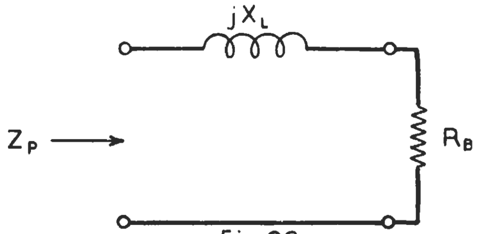


Fig 22

Dealing more specifically with upwards L transducers of the form shown in Fig. 21 (a), consider a case in which the series reactance is connected but the shunt condenser is omitted, as in Fig. 22.

From equations (3) and (4), page 9,

$$Z_P = \frac{R_B^2 + X_L^2}{R_B}//j\left(\frac{R_B^2 + X_L^2}{X_L}\right) \quad \text{............(22)}$$

The equivalent parallel resistance $\dfrac{R_B^2 + X_L^2}{R_B}$ will be greater than R_B and can be made of any required value (above R_B) by suitable adjustment of X_L. Let the value of X_L be such that the equivalent parallel resistance is R_A.

Then,

$$Z_p = R_A \,//\, j\frac{R_B{}^2 + X_L{}^2}{X_L} \quad \text{..........................(23)}$$

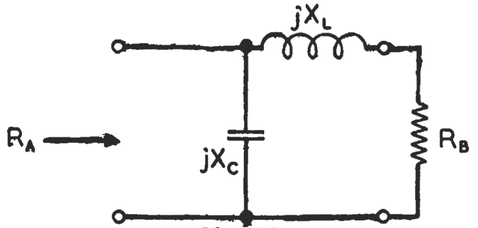


Fig 23

If a shunt condenser is now connected as shown in Fig. 23,

$$Z_p = R_A//j\left(\frac{X\,X_C}{X+X_C}\right) \quad \text{..........................(24)}$$

where $X = \dfrac{R_B{}^2 + X_L{}^2}{X_L}$

If X_C is made equal to $-\left(\dfrac{R_B{}^2 + X_L{}^2}{X_L}\right)$ then,

$$Z_p = R_A$$

Design Formulae. Upwards L Transducer.

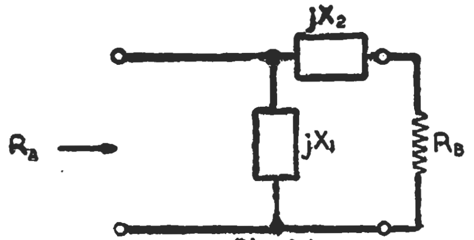


Fig 24

Fig. 24 shows an L-type transducer which is transforming the resistance value of R_B up to a higher value R_A. The required values for X_1 and X_2 can be calculated by the following formulae :—

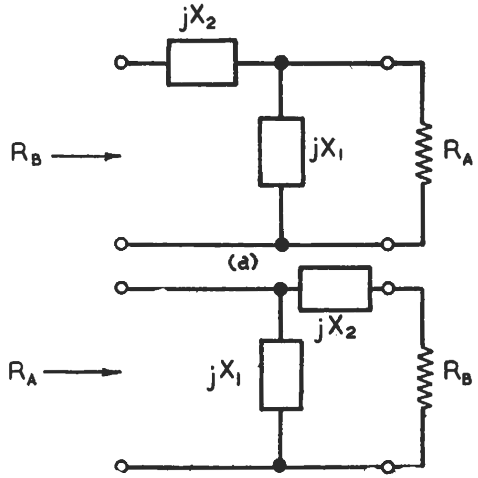
$$X_1 = \pm\frac{mR_B}{\sqrt{m-1}} \quad \text{..................................(25)}$$

$$X_2 = \mp R_B\sqrt{m-1} \quad \text{..............................(26)}$$

where $m = \dfrac{R_A}{R_B}$

The derivation of the above formulae is given in Appendix A, page 48.

General Design Formulae. L-type Transducers.



(b)
Fig 25

It will have been observed that the formulae

$$X_1 = \pm\frac{mR_B}{\sqrt{m-1}}$$

and

$$X_2 = \mp R_B\sqrt{m-1}$$

where $m = \dfrac{R_A}{R_B}$

have been evolved for *both* the downwards L and upwards L transducers. The identity of the formulae for the two cases has come about due to the choice of symbols.

It should be particularly noted that in both cases $m = \dfrac{R_A}{R_B}$, but in the case of the downwards L transducer R_B is the transduced resistance value whereas in the case of the upwards L transducer R_A is the transduced resistance value (see Figs. 25 (a) and 25 (b)).

In treating the above formulae as general design formulae, applicable to either downwards L or upwards L transducers possible confusion will be avoided if the following is noted :—

(i) X_1 is the shunt reactance and X_2 the series reactance.

(ii) m is the ratio of *larger* resistance value to *smaller* resistance value.

Example 10 : Sketch the diagram of an L-type transducer of the type illustrated in Fig. 21 (a), which will transform a resistance of 139 ohms to a resistance of 300 ohms at 1013 kc/s. Mark in L and C values.

Answer:

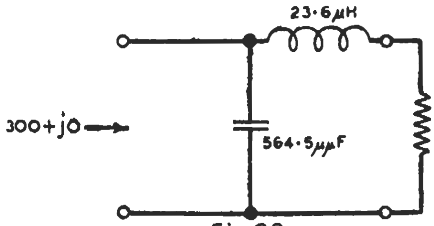


Fig 26

$$M=\frac{300}{139}=2.16$$

$$\sqrt{M-1}=\sqrt{1.16}=1.08$$

$$X_L=+1.08\times 139=+150.1 \text{ ohms}$$

$$X_C=-\frac{2.16\times 139}{1.08}=-278.1 \text{ ohms}$$

$$\therefore\ L=\frac{150.1\times 10^3}{2\pi\times 1013}=23.6\mu H$$

and,

$$C=\frac{10^9}{2\pi\times 1013\times 278.1}=564.5\ \mu\mu F.$$

Example 11: Referring to Example 10, what is the alternative value of L which will produce unity power factor and what will be the value of transformed resistance corresponding with it?

Answer:

In Example 10 the calculated value of X_L is + 150.1. From the information given on page 10, following Example 2,

$$n=\frac{X_L}{R_s}=\frac{150.1}{139}=1.08$$

It follows that the alternative value of X_L is,

$$\frac{R_s}{n}=\frac{139}{1.08}=+128.7$$

$$\therefore\ L=\frac{128.7\times 10^3}{2\pi\times 1013}=20.2\ \mu H$$

The transformed resistance corresponding to this value of inductance is equal to the equivalent parallel resistance of $139+j128.7$

$$=139\ (1+128.7^2/139^2)$$

$$=258.2 \text{ ohms.}$$

The Practical Setting-up of an Upwards L Transducer.

The general remarks made on page 15 in connection with strays, etc., apply equally well to the case of the upwards L transducer.

The practical adjusting of an upwards L transducer, terminated by a resistance R_B, to give the required value of transformed resistance will be considered in relation to three possible cases :—

Case (1). Network of the form shown in Fig. 21 (a). Series inductance variable. Shunt capacity variable.

Case (2). Network of the form shown in Fig. 21 (a). Series inductance variable. Shunt capacity fixed.

Case (3). Network of the form shown in Fig. 21 (b). Series capacity variable. Shunt inductance variable.

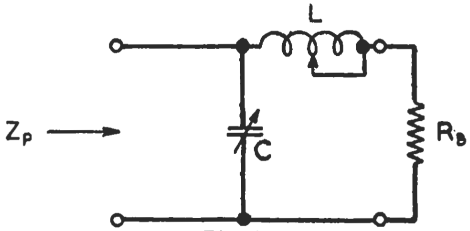


Fig 27

Case (1): Series inductance variable. Shunt capacity variable. Fig. 27.

From the information given on page 16 it will be understood that the correct adjustment of L, Fig. 27, will be such as to make the equivalent parallel resistance component of R_B+jX_L equal to the required value of transformed resistance.

With the bridge connected to measure Z_P, adjust L to provide a resistive component of the measured Z_P equal to the required value of transformed resistance. Then adjust C for unity power factor, i.e. adjust C to the setting that makes the reactance of the measured Z_P infinite.

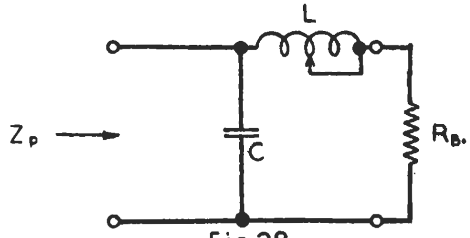


Fig 28

Case (2): Series inductance variable. Shunt capacity fixed. Fig. 28.

This case is the one that has to be dealt with most frequently. The method described above for adjustment of L will not be satisfactory in this case for the reason that the adjustment must now be such as to make the equivalent parallel reactance of R_B+jX_L (Fig. 28) equal and opposite in value to the reactance of C, in order to secure unity power factor.

In the case of the upwards L transducer the effects of varying the inductance in the series arm of the transducer when the shunt capacity is fixed are more complex than in the case of the downwards L transducer.

With the exception of the case where the transformation ratio is 1 : 2 there will be *two* possible values of inductance, either of which will produce zero phase angle (unity power factor) at the terminals of the transducer. Reference to page 10, equations (8) and (9), will make the reason clear.

Since the value of inductance controls the transformation ratio of the network, the two alternative values of L producing unity power factor *will not produce the same value of transformed resistance.* If this fact is remembered, there should be no risk in practice of adjusting the inductance incorrectly.

When there are two possible values of L which will give unity power factor at the terminals of the network the rule indicating whether the smaller or the larger value will be correct is :—If the required transformed resistance value is less than twice the value of the resistance being transformed, then the smaller of the two L values is correct. If the required transformed resistance value is greater than twice the value of the resistance being transformed, then the larger of the two L values is correct.

With the bridge connected to measure Z_P, Fig. 28, adjust L to the value which gives infinite reactance at the input terminals of the network, carefully checking the transformed resistance value to ensure that the wrong alternative setting of L has not been used.

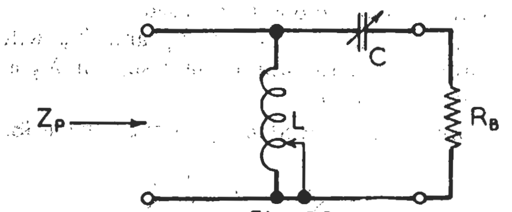


Fig 29

Case (3). Series capacity variable. Shunt inductance variable. Fig. 29.

The schematic of Fig. 29 shows the series arm as consisting of a plain variable condenser, but in practice it is very unlikely to be so. Probably the variable capacitive reactance will be made up of a fixed capacitive reactance (perhaps the reactance of the aerial) in series with a variable inductance. For the moment the simplified equivalent shown in Fig. 29 will suffice.

The series arm of the network should be adjusted first, the correct adjustment being that which makes the resistive component of Z_P equal to the required value of transformed resistance. Then adjust L to give unity power factor.

SECTION F. T-TYPE TRANSFORMING NETWORKS.

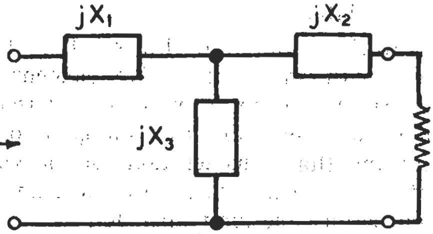


Fig 30

Fig. 30 is a schematic of a transducer of T structure transforming R_1 to R_2. In general, R_2 may be smaller or greater than R_1. It is essential that one of the reactance elements be of opposite sign to the other two.

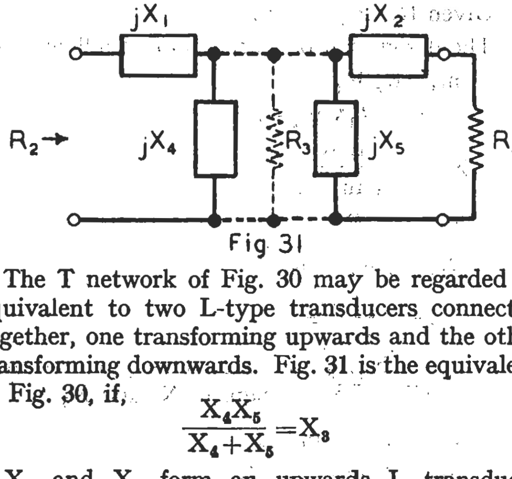


Fig 31

The T network of Fig. 30 may be regarded as equivalent to two L-type transducers connected together, one transforming upwards and the other transforming downwards. Fig. 31 is the equivalent of Fig. 30, if,

$$\frac{X_4X_5}{X_4+X_5}=X_3$$

X_2 and X_5 form an upwards L transducer transforming R_1 up to R_3, where R_3 is the mid-shunt resistance of the T network, while X_4 and X_1 form a downwards L transducer transforming R_3 down to R_2.

T-type transducers are very frequently incorporated in A.T.H. circuits. Using a shunt element of capacitive reactance a T-type transducer is preferable to a simple L-type transducer in any case where the ratio of transformation is near unity and the use of a simple L-type would not provide adequate attenuation of harmonics. Also in cases where the capacitive element is fixed and of an unsuitable value for use in a simple L network, it can often be successfully employed in a T network.

Referring to Fig. 30, the design of the network can be worked out if, in addition to the values of R_1 and R_2, any one of the following are given :—

(*a*) Mid-shunt resistance (R_3).
(*b*) X_2.
(*c*) X_1.
(*d*) X_3.

In the (*d*) case a given value of X_3 will be found to lead to two alternative designs, as will be explained later.

When working out the design of a T-type transducer with shunt capacitive element it is most important to keep in mind the fact that the value of R_3 will determine the voltage across the condenser and that considerations of the voltage rating of the condenser to be used will impose a maximum permissible limit upon R_3.

Design Formulae. T-type Transforming Networks.

The following details regarding the designing of T-type transducers relate to Fig. 30 or its equivalent, Fig. 31.

(*a*) Given R_1, R_2 and R_3,

The design can be worked out as follows :—

$$m_1=R_3/R_1$$
$$X_2=\pm R_1\sqrt{m_1-1}$$
$$X_5=\mp\frac{m_1R_1}{\sqrt{m_1-1}}$$
$$m_2=R_3/R_2$$
$$X_1=\pm R_2\sqrt{m_2-1}$$
$$X_4=\mp\frac{m_2R_2}{\sqrt{m_2-1}}$$
$$X_3=\frac{X_4X_5}{X_4+X_5}$$ due regard being paid to the signs of X_4 and X_5.

(*b*) Given R_1, R_2 and X_2,

$$R_3=R_1\,(1+X_2^2/R_1^2)$$
$$m_1=R_3/R_1$$
$$X_5=\frac{m_1R_1}{\sqrt{m_1-1}}$$ and is opposite in sign to X_2

or,

$$X_5=-X_2\,(1+R_1^2/X_2^2)$$
$$m_2=R_3/R_2$$
$$X_1=\pm R_2\sqrt{m_2-1}$$
$$X_4=\mp\frac{m_2R_2}{\sqrt{m_2-1}}$$
$$X_3=\frac{X_4X_5}{X_4+X_5}$$ due regard being paid to the signs of X_4 and X_5

(*c*) Given R_1, R_2 and X_1,

$$R_3=R_2\,(1+X_1^2/R_2^2)$$
$$m_1=R_3/R_1$$
$$X_2=\pm R_1\sqrt{m_1-1}$$
$$X_5=\mp\frac{m_1R_1}{\sqrt{m_1-1}}$$
$$m_2=R_3/R_2$$
$$X_4=\frac{m_2R_2}{\sqrt{m_2-1}}$$ and is opposite in sign to X_1

or,

$$X_4=-X_1\,(1+R_2^2/X_1^2)$$
$$X_3=\frac{X_4\,X_5}{X_4+X_5}$$ due regard being paid to the signs of X_4 and X_5

(*d*) Given R_1, R_2 and X_3.

This case is one of particular importance as the circumstance frequently occurs that a T network must be designed around some given value of shunt reactance. A given value of X_3 (Fig. 30) gives no indication as to the values of X_4 and X_5 in the equivalent circuit of Fig. 31 and for this reason the case must be treated in a different manner from that employed in (*a*), (*b*) and (*c*).

The values of X_1 and X_2 can be calculated by the following formulae :—

$$X_1=-X_3\left(1\pm\sqrt{\frac{R_2}{R_1}-\frac{R_2^2}{X_3^2}}\right)\quad\text{..............(27)}$$

$$X_2=-X_3\left(1\pm\sqrt{\frac{R_1}{R_2}-\frac{R_1^2}{X_3^2}}\right)\quad\text{..............(28)}$$

It will be seen that each of the above equations has two solutions, which implies that, with given values of R_1 and R_2, there are two alternative networks which could be built around a given value of X_3 and which will produce the required transformation from R_1 to R_2. The mid-shunt resistance values are different in the two cases and practical considerations will invariably dictate which of the two alternatives is the one to be used.

The signs of the reactances X_1 and X_2 will become apparent if the appropriate sign of X_3 is inserted in the formulae.

The formulae are derived in Appendix A, page 48.

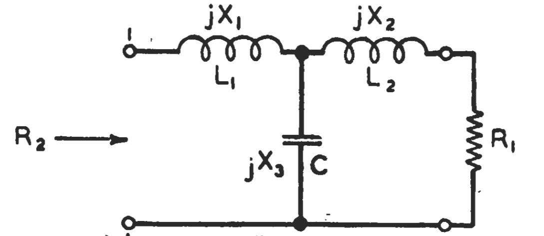


Fig 32

The form normally taken by T-type transducers when employed in A.T.H. circuits is that illustrated in Fig. 32.

Minimum Value of X_3.

The terms under the square root sign in equation (27) must form a positive quantity. Therefore R_2^2/X_3^2 must not be greater than $\frac{R_2}{R_1}$, from which it follows that the minimum value of X_3 is $\pm\sqrt{R_1 R_2}$.

Example 12 : Design a T network to transform a resistance of 100 ohms to a resistance of 80 ohms, the mid-shunt resistance to be 700 ohms. The frequency is 668 kc/s and the network is of the form illustrated in Fig. 32.

Answer : Refer to method (*a*), page 20.

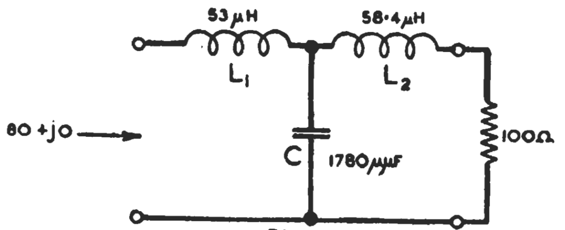


Fig. 33

$$m_1=\frac{700}{100}=7$$
$$\sqrt{m_1-1}=2.45$$
$$X_2=+2.45\times100=+245 \text{ ohms}$$
$$\therefore\ L_2=\frac{245\times10^3}{2\pi\times668}=58.4\ \mu\text{H}$$
$$X_5=-\frac{7\times100}{2.45}=-286 \text{ ohms}$$
$$m_2=\frac{700}{80}=8.75$$
$$\sqrt{m_2-1}=2.78$$
$$X_1=+2.78\times80=+222.4 \text{ ohms}$$
$$\therefore\ L_1=\frac{222.4\times10^3}{2\pi\times668}=53\ \mu\text{H}$$
$$X_4=-\frac{8.75\times80}{2.78}=-251 \text{ ohms}$$
$$X_3=-\frac{-286\times-251}{-286\ -\ 251}=-133.7 \text{ ohms}$$
$$\therefore\ C=\frac{10^9}{2\pi\times668\times133.7}=1780\ \mu\mu\text{F}$$

Example 13 : Calculate the reactance and mid-shunt resistance values for a T transducer to transform a resistance of 250Ω to a resistance of 300Ω, given that the shunt arm has a reactance of −500. Deal with both alternative networks.

Answer:

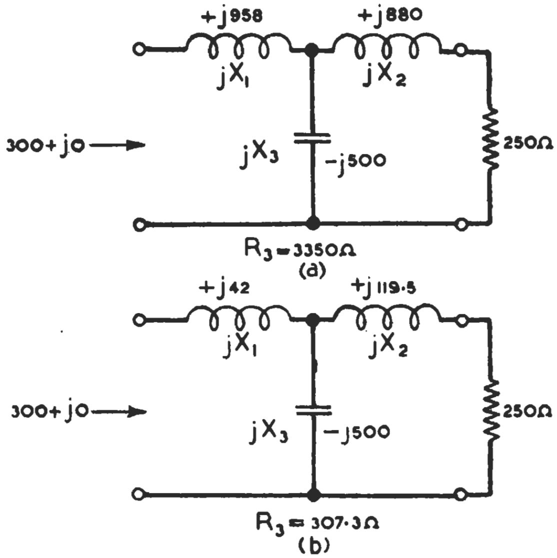


Fig 34.

By equation (27), and giving X_3 its appropriate sign,

$$X_1=+500\left[1\pm\sqrt{\frac{300}{250}-\frac{300^2}{(-500)^2}}\right]$$
$$=+500\,(1\pm\sqrt{1.2-0.36})$$
$$=+500\,(1\pm0.916)$$
$$=+958 \text{ or } +42.0 \text{ ohms}$$

By equation (28),

$$X_2=+500\left[1\pm\sqrt{\frac{250}{300}-\frac{250^2}{(-500)^2}}\right]$$
$$=+500\,(1\pm\sqrt{0.83-0.25})$$
$$=+500\,(1\pm0.761)$$
$$=+880 \text{ or } +119.5 \text{ ohms}$$

The two alternative networks are illustrated in Figs. 34 (a) and 34 (b) respectively.

The mid-shunt resistance of the network of Fig. 34 (a) equals the equivalent parallel resistance of 250+j880 ohms.

$$\therefore\ R_3=250\,(1+880^2/250^2)$$
$$=3350 \text{ ohms}$$

In the case of Fig. 34 (b), the mid-shunt resistance equals the equivalent parallel resistance of 250+j119.5 ohms.

$$R_3=250\,(1+119.5^2/250^2)$$
$$=307.3 \text{ ohms}$$

The Practical Setting-up of a T-type Transducer of the Form illustrated in Fig. 32.

Case 1. Shunt capacity fixed. Inductance arms variable.

With fixed values of R_1 and C, both the transformed resistance and the mid-shunt resistance are dependent upon the setting of L_2. When setting up the network care must be taken that L_2 is not adjusted to a value which represents a wrong choice of the two alternative values. Sometimes the value of L_2 is, in itself, a sufficiently obvious indication, but if there is the slightest doubt on the matter a check should be made on the mid-shunt resistance value.

To set up the transducer the bridge should be connected to the terminals 1, 1 (Fig. 32). L_2 should be adjusted until the series resistive component of the measured impedance is equal to the desired value of transformed resistance. It is at this stage that the mid-shunt resistance can be checked if desired. All that is necessary in this connection is to cut out L_1 temporarily and then ascertain on the bridge the value of the parallel resistive component of the measured impedance.

Finally, adjust L_1 for unity power factor, i.e. infinite parallel reactance.

Case 2. Shunt capacity variable. Inductance arms variable.

In this case the mid-shunt resistance is dependent upon the setting of L_2, but the transformed resistance is dependent upon the settings of both L_2 and C.

With the bridge connected to terminals 1, 1 (Fig. 32) and with L_1 cut out of circuit, L_2 should be adjusted until a value of the parallel resistance component of the measured impedance approximating to that calculated for the mid-shunt resistance is obtained. Now adjust C until the series resistive component of the measured impedance is equal to the required transformed resistance value. Finally, connect in L_1 and adjust it until unity power factor is obtained.

Quarter-wave T Network.

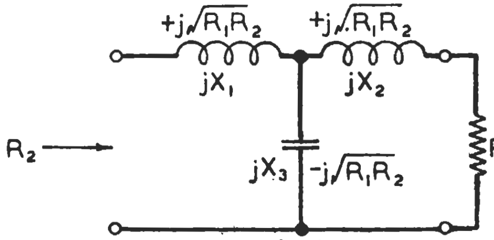


Fig 35

In the transducer of Fig. 35 let $X_3 = -\sqrt{R_1R_2}$.
Then, by equation (27), page 20,

$$X_1 = \sqrt{R_1R_2}\left(1 \pm \sqrt{\frac{R_2}{R_1} - \frac{R_2^2}{R_1R_2}}\right) = \sqrt{R_1R_2}$$

and, by equation (28),

$$X_2 = \sqrt{R_1R_2}\left(1 \pm \sqrt{\frac{R_1}{R_2} - \frac{R_1^2}{R_1R_2}}\right) = \sqrt{R_1R_2}$$

Thus the transducer has a symmetrical construction with each arm having a reactance of $\pm\sqrt{R_1R_2}$ ohms.

If a length of feeder equal to a quarter-wavelength (or an odd multiple of a quarter-wavelength) has a characteristic impedance $R_0 = \sqrt{R_1R_2}$ and is terminated by R_1, then the impedance looking into the other end of the feeder is R_2. Thus, from the transformation point of view, the network illustrated in Fig. 35 behaves similarly

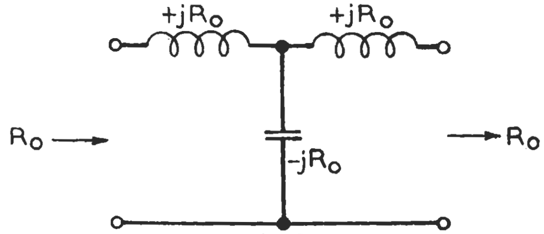


Fig 36

to a quarter-wavelength of feeder of characteristic impedance $R_0 = \sqrt{R_1R_2}$.

Let $R_1 = R_2 = R_0$ so that $\sqrt{R_1R_2} = R_0$.

In such a case the network will take the form illustrated in Fig. 36 and with a termination of R_0 at either end of the network, the impedance looking into the other end will be R_0. This network will be equivalent to a quarter-wavelength of feeder of characteristic impedance R_0. Thus, if desired, it could be used to " build out " the length of a feeder of characteristic impedance R_0, being equivalent to the addition of a quarter-wavelength.

Example 14: Show that the mid-shunt resistance of a quarter-wave T network transforming R_1 to R_2 is equal in value to R_1 and R_2 in series.

Answer:

Referring to Fig. 35, the equivalent parallel resistance of $R_1 + jX_2$,

$$= R_1(1 + X_2^2/R_1^2)$$

$$= R_1\left(1 + \frac{R_1R_2}{R_1^2}\right)$$

$$= R_1\left(\frac{R_1 + R_2}{R_1}\right)$$

$$= R_1 + R_2$$

SECTION G. π-TYPE TRANSFORMING NETWORKS.

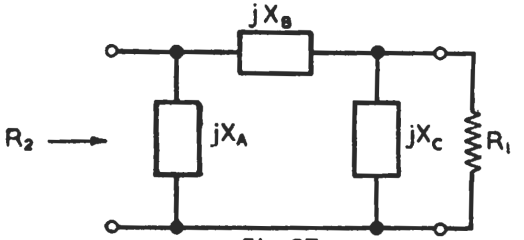


Fig 37

Fig. 37 is a schematic of a transforming network of π structure, transforming R_1 to R_2. It is essential that one of the reactance elements be of opposite sign to the other two.

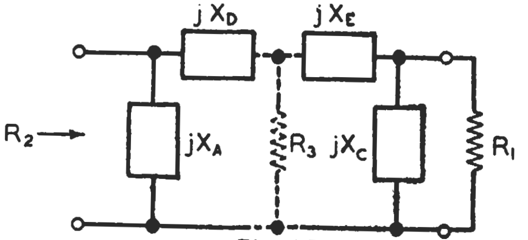


Fig 38

As in the case of the T-type transducer, the π-type transducer can be regarded as equivalent to two L-type transducers, as illustrated in Fig. 38. In the networks of Fig. 38.

$$X_D+X_E=X_B$$

X_C and X_E form a downwards L transducer transforming R_1 down to R_3, while X_D and X_A form an upwards L transducer transforming R_3 up to R_2.

Design Formulae. π-type Transducers.

(*a*) Given R_1, R_2 and R_3,

$$m_1=R_1/R_3$$
$$X_C=\pm\frac{m_1R_3}{\sqrt{m_1-1}}$$
$$X_E=\mp R_3\sqrt{m_1-1}$$
$$m_2=R_2/R_3$$
$$X_D=\mp R_3\sqrt{m_2-1}$$
$$X_A=\pm\frac{m_2R_3}{\sqrt{m_2-1}}$$
$$X_B=X_D+X_E$$

(*b*) Given R_1, R_2 and X_C,

$$R_3=\frac{R_1}{1+R_1^2/X_C^2}$$
$$m_1=R_1/R_3$$

$X_E=R_3\sqrt{m_1-1}$ and is opposite in sign to X_C

or

$$X_E=\frac{-X_C}{1+X_C^2/R_1^2}$$
$$m_2=R_2/R_3$$
$$X_D=\pm R_3\sqrt{m_2-1}$$
$$X_A=\mp\frac{m_2R_3}{\sqrt{m_2-1}}$$
$$X_B=X_D+X_E$$

(*c*) Given R_1, R_2 and X_A,

$$R_3=\frac{R_2}{1+R_2^2/X_A^2}$$
$$m_1=R_1/R_3$$
$$X_C=\pm\frac{m_1R_3}{\sqrt{m_1-1}}$$
$$X_E=\mp R_3\sqrt{m_1-1}$$
$$m_2=R_2/R_3$$

$X_D=R_3\sqrt{m_2-1}$ and is opposite in sign to X_A

or

$$X_D=\frac{-X_A}{1+X_A^2/R_2^2}$$
$$X_B=X_D+X_E$$

(*d*) Given R_1, R_2 and X_B,

$$X_A=\frac{-R_2X_B}{R_2\pm\sqrt{R_1R_2-X_B^2}} \quad \text{..............(29)}$$

$$X_C=\frac{-R_1X_B}{R_1\pm\sqrt{R_1R_2-X_B^2}} \quad \text{..............(30)}$$

These formulae are derived in Appendix A, page 49.

Each equation has two solutions. Thus there are two alternative networks which can be built around a given value of X_3 and which will both provide the same transformation.

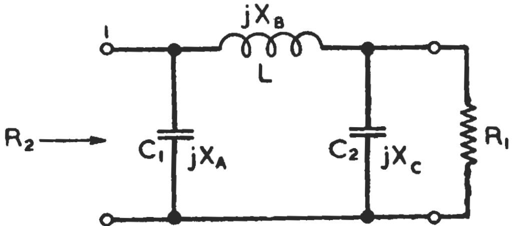


Fig 39

The form in which the π-type transducer is usually to be found when employed in A.T.H. circuits is as illustrated in Fig. 39. It will be seen that the network is of low-pass filter character. The use of such a network is of particular advantage when attenuation of harmonics is important.

***Example* 15: Calculate the reactance values for a π-type transducer of the form illustrated in Fig. 39, given that R_1=250 ohms, R_2=300 ohms and X_C=−212 ohms.**

Answer:

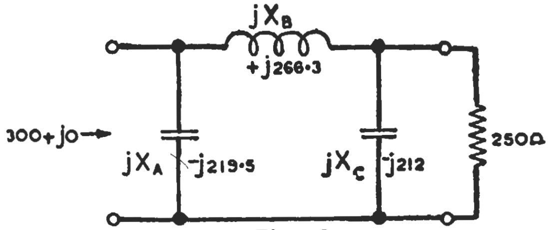


Fig 40

Refer to method (*b*),

$$R_3=\frac{250}{1+250^2/(-212)^2}=104.6 \text{ ohms}$$

$$m_1=\frac{250}{104.6}=2.391$$

$$\sqrt{m_1-1}=1.18$$

$X_E=1.18\times104.6=123.3$ and is of positive sign, since X_B is negative

$$m_2=\frac{300}{104.6}=2.869$$

$$\sqrt{m_2-1}=1.368$$

$X_D=1.368\times104.6=143.0$ and is of positive sign, since X_A is negative

$$X_A=-\frac{2.869\times104.6}{1.368}=-219.5 \text{ ohms}$$

$$X_B=+143.0+123.3=+266.3 \text{ ohms}$$

The Practical Setting-up of a π-type Transducer of the Form illustrated in Fig. 39.

Case 1. Series inductance variable. Shunt capacity arms, both fixed.

There are two possible values of L (Fig. 39) which will produce unity power factor at the terminals 1, 1, but the transformed resistance values are different.

The bridge should be connected to terminals 1, 1 and L adjusted, first, with the object of getting the parallel resistive component of the measured impedance to approximate to the required value of transformed resistance and, finally, to obtain unity power factor.

When the value of R_p is close to the required transformed resistance value, the phase angle will be small, so that the final variation of L to obtain unity power factor will be a matter of fine adjustment.

Case 2. Series inductance variable. C_2 fixed. C_1 variable.

With the bridge connected to terminals 1, 1, L should be adjusted until the parallel resistive component of the measured impedance is equal to the required transformed resistance value. Then C_1 should be adjusted to obtain unity power factor.

Case 3. Series inductance variable. Capacity arms both variable.

With the bridge connected directly across C_2 and L disconnected, the former should be adjusted until the series resistive component of the measured impedance is equal to the calculated value of mid-series resistance.

Then proceed as for Case 2.

Quarter-wave π Network.

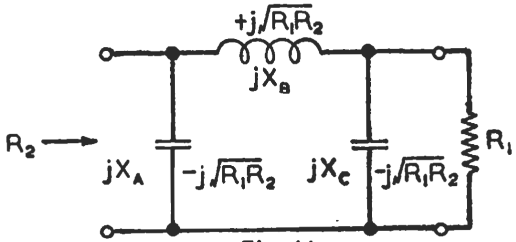


Fig 41

In the transducer illustrated in Fig. 41, let $X_B=+\sqrt{R_1R_2}$.

Then, by equations (29) and (30),

$$X_A=\frac{-R_1\sqrt{R_1R_2}}{R_1\pm\sqrt{R_1R_2-R_1R_2}}=-\sqrt{R_1R_2}$$

$$X_C=\frac{-R_2\sqrt{R_1R_2}}{R_2\pm\sqrt{R_1R_2-R_1R_2}}=-\sqrt{R_1R_2}$$

Thus the network has a symmetrical construction, each arm having a reactance of $\pm\sqrt{R_1R_2}$ ohms.

As in the case of the equivalent T-type network (Fig. 36), the network illustrated in Fig. 41 is equivalent to a quarter-wave feeder line of characteristic impedance $R_o=\sqrt{R_1R_2}$.

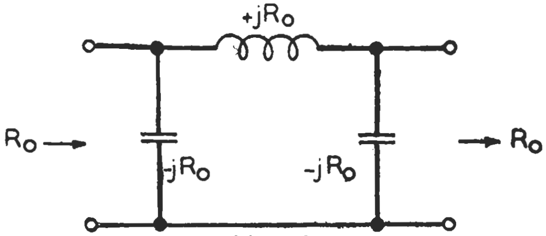


Fig 42

Fig. 42 illustrates a quarter-wave π-type network equivalent to a quarter-wave feeder line of characteristic impedance R_o.

Example 16: Show that the mid-series resistance of a quarter-wave π network transforming

R_1 to R_2 is equal in value to R_1 and R_2 in parallel.

Answer :

Referring to Fig. 41, the equivalent series resistance of $R_1//jXc$,

$$=\frac{R_1Xc^2}{R_1^2+Xc^2}$$

$$=\frac{R_1^2R_2}{R_1^2+R_1R_2}$$

$$=\frac{R_1R_2}{R_1+R_2}$$

SECTION H. A.T.H. CIRCUITS.

The representative aerial impedance curves given in Figs. 6 and 7 relate to the *series* equivalent aerial impedance. To any particular impedance of the form $R+jX_s$ there is a *parallel* equivalent, $R_p//jX_p$, and it must be understood that, despite the fact that aerial impedance curves normally give information regarding the series aspect only, the aerial impedance may equally well be expressed in parallel terms. Thus, if the aerial impedance happens to be $Z_s=50+j200$ ohms, it is equally true to state that the impedance is $Z_p=850//212$ ohms, where Z_p is the parallel equivalent of Z_s.

The first part of this section will be devoted to those cases where matching has to be secured between the impedance of an aerial and the characteristic impedance of an unbalanced feeder.

Matching Networks between Aerial and Unbalanced Feeder.

The choice of network to be used in any particular case is frequently determined by such practical considerations as the number and nature of the components which are available and under war-time conditions this sometimes makes the latitude of choice very restricted.

For the purpose of making the subject of alternative networks clear, however, the matter will first be considered without reference to possible practical restrictions. Later in this section examples with more practical significance will be dealt with.

Given the driving-point impedance of an aerial and the characteristic impedance, (R_o), of the feeder to which it is to be matched, general design outlines from which the practical design may be chosen can be made as follows :—

(1) Taking the *series* aspect of the aerial impedance, sketch a network which will transform R_s to R_o, R_s being the equivalent series resistance of the aerial impedance and R_o the characteristic impedance of the feeder.

(2) Consider X_s, the equivalent series reactance of the aerial, and decide whether it can be incorporated into the network as part of the reactance of the latter, or whether it must be balanced out.

(3) Sketch the complete network embodying (1) and (2).

(4) Taking the *parallel* aspect of the aerial impedance, sketch a network which will transform R_p to R_o.

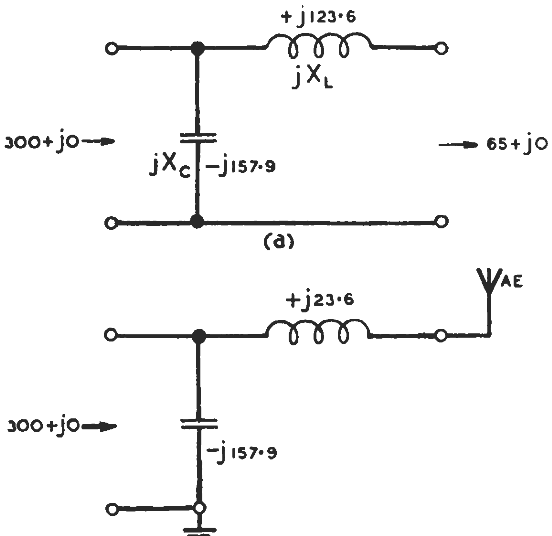
(5) Consider X_p on the same lines as in (2).

(6) Sketch the complete network.

In the following four examples illustrating the above only simple L networks will be dealt with. Where π or T networks are concerned there are, of course, limitless theoretical possibilities, but the use of such networks will be considered in examples of more practical significance.

Example 17 : Consider design outlines for a case where the aerial impedance is $Z_s=65+j100$ ohms and the feeder characteristic impedance is $R_o=300$ ohms.

Answer :



(b)
Fig 43

(1) A network that will transform $R_s=65$ ohms to $R_o=300$ ohms is illustrated in Fig. 43 (a).

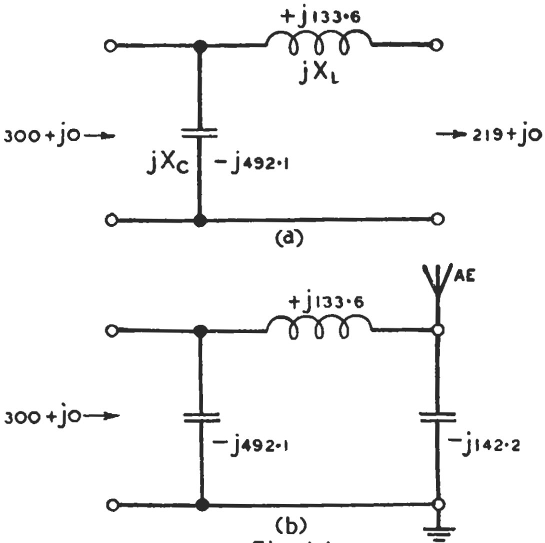
$$m=\frac{300}{65}=4.62 \quad \sqrt{m-1}=1.902$$

$$X_L=+1.902\times 65 \ =+123.6 \text{ ohms}$$

$$X_C=-\frac{4.62\times 65}{1.902}=-157.9 \text{ ohms}$$

(2) It will be seen that transformation from 65 ohms to 300 ohms demands a series reactance of +123.6 ohms. The equivalent series reactance of the aerial is +100 ohms and this can form part of the transforming network.

(3) The complete network is illustrated in Fig. 43 (b). The series reactance coil has a value of +23.6 ohms which, with the aerial reactance, provides the +123.6 ohms required for transformation.



(a)

(b)
Fig 44

(4) The equivalent parallel impedance of the aerial is $Z_p=219//j142.2$ ohms.

A network transforming $R_p=219$ ohms to $R_o=300$ ohms is illustrated in Fig. 44 (a)

$$m=\frac{300}{219}=1.371 \quad \sqrt{m-1}=0.61$$

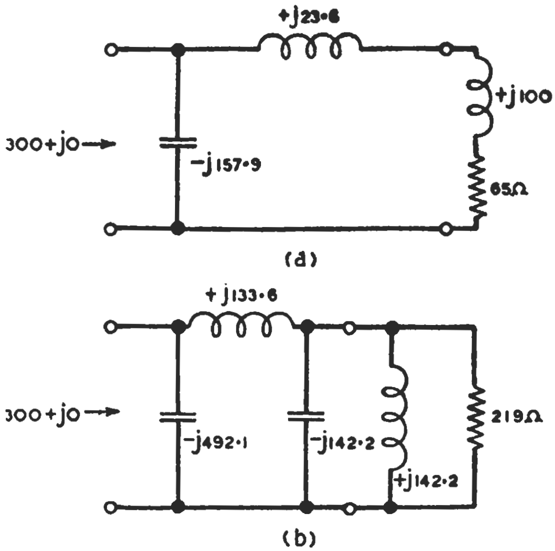
$$X_L=+0.61\times 219=+133.6 \text{ ohms}$$

$$X_C=-\frac{1.371\times 219}{0.61}=-492.1 \text{ ohms}$$

(5) The equivalent parallel reactance of the aerial, namely, +142.2 ohms, cannot be incorporated into the transforming network, so it will be necessary to balance it out, which can be done by a parallel reactance of −142.2 ohms.

(6) The complete network is illustrated in Fig. 44 (b).

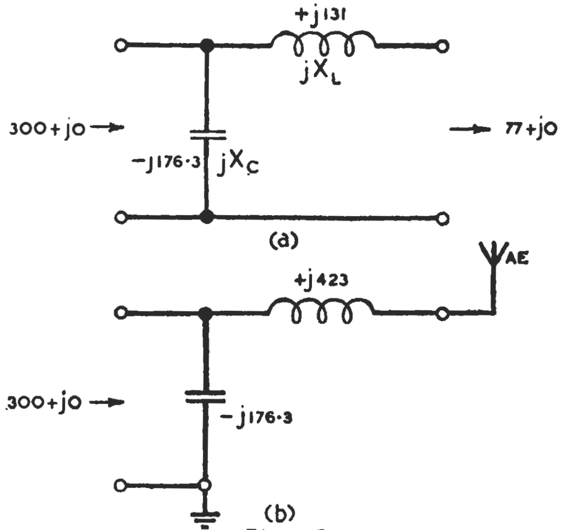
The functioning of A.T.H. circuits may not always be immediately apparent by inspection of circuit diagrams of the type of Figs. 43 (b) and 44 (b). In such cases it will prove helpful if the aerial impedance components are sketched in, using the equivalent series or equivalent parallel forms. Thus, Fig. 45 (a) is the equivalent of Fig. 43 (b) and Fig. 45 (b) is the equivalent of Fig. 44 (b).



(a)

(b)
Fig 45

Example 18: On the lines of Example 17, deal with the case where the aerial impedance is $Z_s=77-j292$ ohms. R_o of feeder=300 ohms.



(a)

(b)
Fig 46

Answer :

(1) A network that will transform $R_s=77$ ohms to 300 ohms is illustrated in Fig. 46 (a).

$$m=\frac{300}{77}=3.9 \quad \sqrt{m-1}=1.704$$

$$X_L=+1.704\times 77=+131 \text{ ohms}$$

$$X_C=-\frac{3.9\times 77}{1.704}=-176.3 \text{ ohms}$$

(2) The aerial reactance is -292, which can be balanced out by a series reactance of $+292$ ohms.

(3) A reactance of $+131$ is required for the resistance transforming network and a reactance of $+292$ ohms to balance out the aerial reactance, making a total of $+423$ ohms. The complete network is illustrated in Fig. 46 (b).

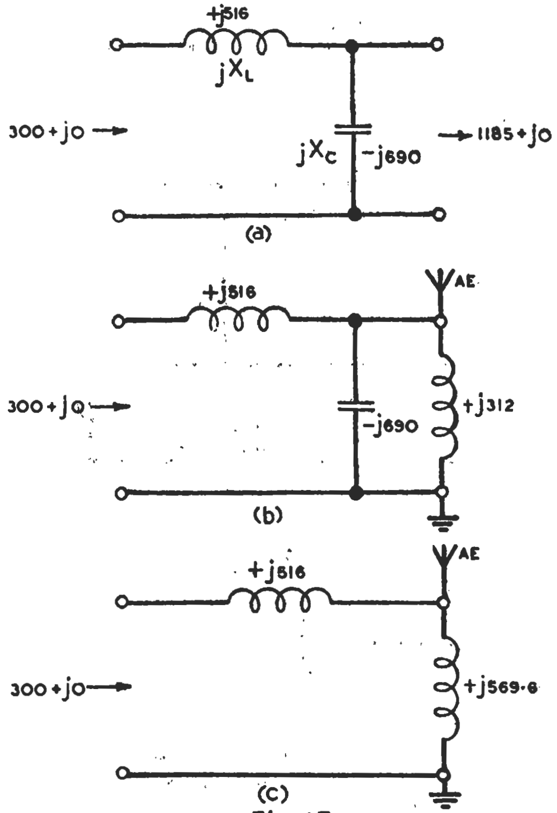


Fig 47

(4) The equivalent parallel impedance of the aerial is $Z_p=1185//-j312$ ohms.

A network transforming $R_p=1185$ ohms to $R_o=300$ ohms is illustrated in Fig. 47 (a).

$$m=\frac{1185}{300}=3.95 \quad \sqrt{m-1}=1.72$$

$$X_L=+1.72\times300=+516 \text{ ohms}$$

$$X_C=-\frac{3.95\times300}{1.72}=-690 \text{ ohms}$$

(5) It will be seen that the parallel reactance of the aerial is smaller than the value of X_C required. The aerial reactance could be balanced out by a parallel reactance of $+312$ ohms, but, alternatively, the negative aerial reactance could be shunted by a positive reactance of such value that the combination of the two is equivalent to the desired reactance value of -690 ohms. If a reactance X_C is shunted by a reactance X_L the total reactance is, of course,

$$\frac{X_LX_C}{X_L+X_C}$$

Thus, in the case under consideration, what is required is a value of X_L such that,

$$\frac{-312X_L}{X_L-312}=-690$$

Solving this equation for X_L,

$$X_L=+569.6 \text{ ohms.}$$

(6) Fig. 47 (b illustrates the complete network in the case where the parallel reactance of the aerial is balanced out. Fig. 47 (c) illustrates the alternative case where a shunt reactance of $+569.6$ ohms is provided for the purpose of increasing the -312 ohms aerial reactance up to the -690 ohms required for transformation.

Fig. 47 (c) shows the reactances of $+516$ and $+569.6$ ohms respectively in the form of two separate coils which are not coupled together.

An arrangement of simpler construction, but one that is not so easy to adjust would be a circuit embodying one coil only, with the aerial tapped on to it. Owing to the effects of mutual inductance between the sections above and below the aerial tapping, adjustments by trial and error methods will be necessary. These adjustments will involve varying the number of turns included between the aerial tap and earth, and also the number of turns in the section above the aerial tap.

It should be noted that the circuit of Fig. 47 (c) (or the alternative described in the previous paragraph), is not a satisfactory circuit to use when attenuation of harmonics in the network is important.

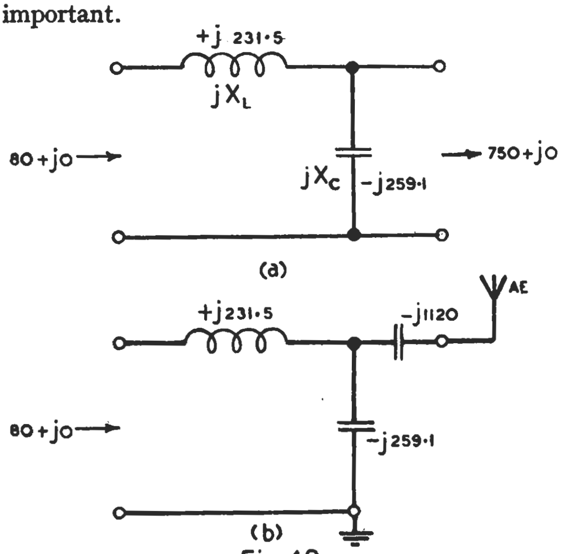


Fig 48

Example 19: On the lines of the preceding examples, deal with $Z_s=750+j1120$ ohms. $R_o=80$ ohms.

Answer:

(1) A network transforming $R_s=750$ ohms to $R_o=80$ ohms is illustrated in Fig. 48 (a).

$$m=\frac{750}{80}=9.375 \quad \sqrt{m-1}=2.894$$

$$X_L=+2.894\times 80=+231.5 \text{ ohms}$$

$$X_C=-\frac{9.375\times 80}{2.894}=-259.1 \text{ ohms}$$

(2) The equivalent series reactance of the aerial cannot be incorporated into the transducer of Fig. 48 (b), so must be balanced out.

This can be done by a series reactance of -1120 ohms.

(3) Fig. 48 (b) shows the complete network.

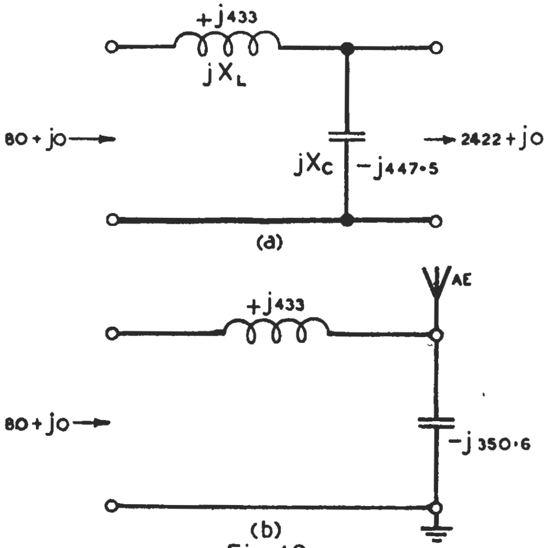


Fig 49

(4) The equivalent parallel impedance of the aerial is,

$$Z_p=2422//j1623 \text{ ohms}$$

Fig. 49 (a) illustrates the network which will transform $R_p=2422$ ohms to $R_o=80$ ohms.

$$m=\frac{2422}{80}=30.275 \quad \sqrt{m-1}=5.412$$

$$X_L=+5.412\times 80=+433 \text{ ohms}$$

$$X_C=-\frac{30.275\times 80}{5.412}=-447.5 \text{ ohms}$$

(5) The aerial reactance is of the wrong sign to be incorporated into the transducer of Fig. 49 (a), but it can be balanced out by a parallel reactance of -1623 ohms. A condenser used for this purpose would be in parallel with the capacity of the transforming network, so the requirements of (*a*) balancing out aerial reactance and (*b*) transformation could be fulfilled by a single parallel reactance of

$$-\left(\frac{1623\times 447.5}{1623+447.5}\right)=-350.6 \text{ ohms}$$

(6) Fig. 49 (b) illustrates the complete network.

Example 20: On the lines of the preceding examples, deal with $Z_s=1800-j600$ ohms. $R_o=300$ ohms.

Answer:

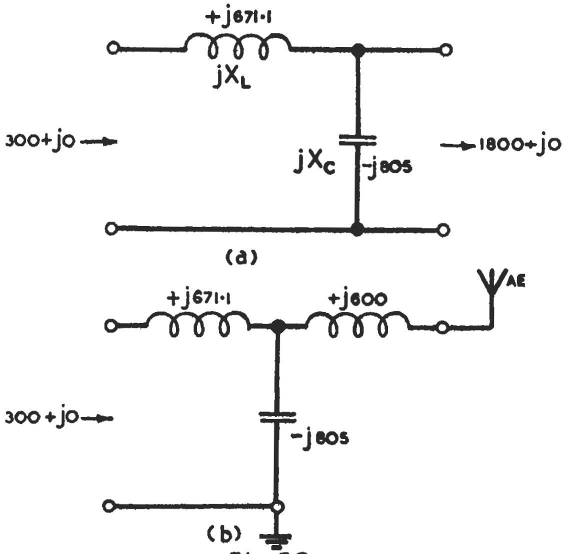


Fig 50

(1) A network which will transform $R_s=1800$ ohms to $R_o=300$ ohms is illustrated in Fig. 50 (a).

$$m=\frac{1800}{300}=6 \quad \sqrt{m-1}=2.237$$

$$X_L=+2.237\times 300=+671.1 \text{ ohms}$$

$$X_C=-\frac{6\times 300}{2.237}=-805 \text{ ohms}$$

(2) The equivalent series reactance of the aerial cannot be incorporated into the transforming network of Fig. 50 (a), but can be balanced out by a series reactance of $+600$ ohms.

(3) The complete network is illustrated in Fig. 50 (b).

(4) The equivalent parallel impedance of the aerial is,

$$Z_p=2000//-j6000 \text{ ohms}$$

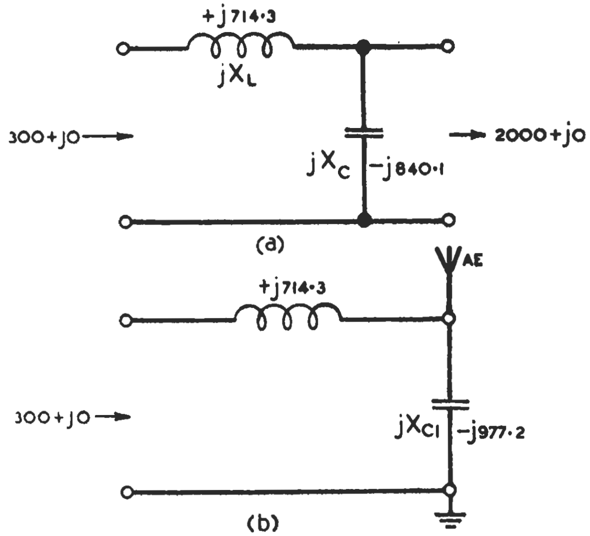


Fig 51

A network which will transform R_p=2000 ohms to R_o=300 ohms is illustrated in Fig. 51 (a).

$$m=\frac{2000}{300}=6.667 \quad \sqrt{m-1}=2.381$$

$$X_L=+2.381\times300=+714.3 \text{ ohms}$$

$$X_C=-\frac{6.667\times300}{2.381}=-840.1 \text{ ohms}$$

(5) The aerial reactance is larger than the shunt reactance required by the transducer, but the correct reactance can be obtained by employing a shunt reactance, X_{C_1}, such that

$$\frac{-6000X_{c_1}}{X_{c_1}-6000}=-840.1$$

Solving the above for X_{c_1} gives,
$X_{c_1}=-977.2$ ohms

(6) The complete network is illustrated in Fig. 51 (b).

It should be noted that it does not follow that the alternative circuits considered in Examples 17-20 necessarily include the circuits that would be employed in practice. When it comes to the question of deciding upon the most suitable network to use in any particular case hard and fast rules cannot be laid down. The problem must, in every case, be weighed up with due regard to the particular practical circumstances associated with it. The following notes are given to serve as a general guide, and it should be understood that in practice some of the considerations may prove to be conflicting :—

(*a*) In the interests of economy every consideration should be given to the possibility of using the simplest of the alternative set-ups and the one involving the smallest number of components.

(*b*) In cases where transmission takes place on two or more alternative frequencies consideration should be given to the designing of the A.T.H. circuits in such manner as to make the change-over as simple as possible.

(*c*) In Examples 17-20 the frequency of transmission was not specified, so no consideration was given to the values of inductance and capacity required by the networks. In practice, however, this is a very vital consideration and it will often be found that modifications of design must be made simply because the components available are not of values that can be used in a contemplated design. Particularly is such a situation liable to arise in connection with condensers of fixed capacity.

(*d*) The voltage and current ratings of the available components must receive the closest attention and network design should include the approximate calculation of voltage and current, and their comparison with the ratings of the available components.

(*e*) In greater or less degree, according to circumstances, the matter of the attenuation of harmonic frequencies is important and, as already stated, is responsible for the common use of networks employing series inductance and shunt capacity. In this connection a set-up such as that of Fig. 47 (c) is frequently to be ruled out, although it would be possible to use it if the transmitter output has negligible harmonic content.

(*f*) It is essential that A.T.H. networks should not cause any sideband cutting. It is very unlikely that any particular modification of design will be required on this account where M.W. networks are concerned, but it should be noted that the transformation ratio of any L-type combination should not be allowed to exceed 100.

(*g*) It is important to note that whenever a series condenser is connected in such a circuit position as to insulate the aerial from earth it is necessary to employ a static leak.

Before dealing with the subject of current and voltage calculations, the following Example is given to illustrate a typical case involving considerations of inductance and capacity values.

Example 21 : The equivalent series impedance of an aerial is $Z_s = 15 - j170$ ohms, and it is to be matched to an unbalanced feeder with a characteristic impedance of $R_c = 80$ ohms. The carrier frequency is 668 kc/s.

(*a*) Draw the circuit and calculate L and C values for a matching network based upon the series aspect of the aerial impedance. Use a simple L-type network.

(*b*) Assuming that a condenser of 4000 $\mu\mu$F is the only condenser available, draw the circuit of a suitable matching network employing this value of capacity.

Answer (*a*) :

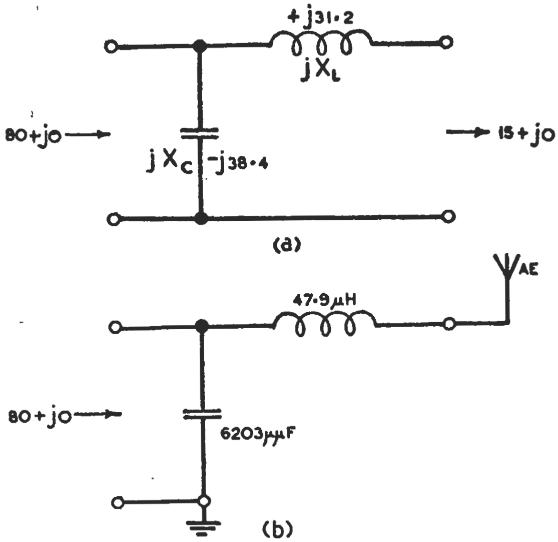


Fig 52

The resistance transforming network is illustrated in Fig. 52 (a).

$$m = \frac{80}{15} = 5.333 \quad \sqrt{m-1} = 2.082$$

$$X_L = +2.082 \times 15 = +31.2 \text{ ohms}$$

$$X_C = -\frac{5.333 \times 15}{2.082} = -38.4 \text{ ohms}$$

The aerial reactance of -170 can be balanced out by a series reactance of $+170$. Thus a total inductive reactance of $31.2 + 170 = 201.2$ ohms is required.

$$L = \frac{201.2 \times 10^3}{2\pi \times 668} = 47.9\ \mu\text{H}$$

$$C = \frac{10^9}{2\pi \times 668 \times 38.4} = 6203\ \mu\mu\text{F}$$

The complete network is illustrated in Fig. 52 (b).

Answer. (*b*)

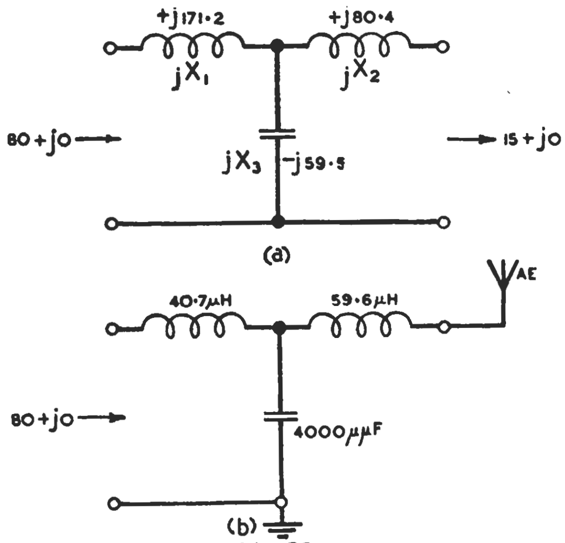


Fig 53

The reactance of 4000 $\mu\mu$F at 668 kc/s is

$$-\frac{10^9}{2\pi \times 668 \times 4000} = -59.5 \text{ ohms}$$

This reactance can be employed in a T-type transducer as shown in Fig. 53 (a).

By equation (27), page 20,

$$X_1 = 59.5 \left[1 \pm \sqrt{\frac{80}{15} - \frac{80^2}{(-59.5)^2}} \right]$$

Taking the positive square root (in order that X_1 shall be a positive reactance),

$$X_1 = +171.2 \text{ ohms}$$

The inductance value required is,

$$\frac{171.2 \times 10^3}{2\pi \times 668} = 40.7\ \mu\text{H}$$

By equation (28),

$$X_2 = 59.5 \left[1 \pm \sqrt{\frac{15}{80} - \frac{15^2}{(-59.5)^2}} \right]$$

Taking the positive square root,

$$X_2 = +80.4 \text{ ohms}$$

To this reactance must be added $+170$, required to balance out the aerial reactance, making a total of $+250.4$ ohms.

The inductance value required is,

$$\frac{250.4 \times 10^3}{2\pi \times 668} = 59.6\ \mu\text{H}$$

The complete network is illustrated in Fig. 53 (b).

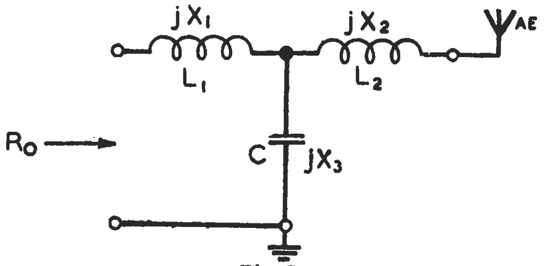


Fig 54

A.T.H. matching networks of the form illustrated in Fig. 54 are very commonly used, but it should be noted that from the functioning point of view there are a number of circuit varieties. These are summarised below :

(*a*) Aerial impedance $Z_s=R_s+jX_s$ and X_s is negative. The inductance L_2 is adjusted to balance out the series aerial reactance. The condenser C is the shunt arm of a simple L-type transforming network.

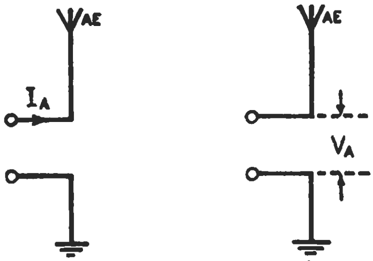
(*b*) Aerial impedance $Z_s=R_s+jX_s$ and X_s is negative. The inductance L_2 is adjusted to a value greater than that required to balance out the series aerial reactance. The condenser C is the shunt arm of a T-type transforming network of which L_1 is one of the series arms and *part* of L_2 the other.

(*c*) Aerial impedance $Z_s = R_s + jX_s$ and X_s is negative. The inductance L_2 is adjusted to a value less than that required to balance out the series aerial reactance, whilst C and a capacity corresponding to the equivalent parallel reactance of $R_s+j(X_s+X_2)$ together form the shunt arm of an L-type transforming network. In this circuit the voltage across C is less than the voltage to ground at the aerial driving point. This circuit considered as a modification of the simple parallel circuit, has useful application in certain cases where the condenser voltage would otherwise be too high.

(*d*) Aerial impedance $Z_s=R_s+jX_s$ and X_s is positive. In this case, whatever the value of L_2, C can be regarded as equivalent to two capacities in parallel, one balancing out the equivalent parallel reactance of $R_s+j(X_s+X_2)$ and the other forming the shunt arm of an L-type transducer. In this circuit the voltage across the condenser will be greater than the voltage to ground at the aerial driving point.

Estimating Voltages and Currents.

In making (approximate) calculations of the voltages and currents in an A.T.H. network, the network components are regarded as loss-free.



(a) Fig 55 (b)

The basis of the calculations is normally the value of carrier R.F. power required in the aerial. Given this value and also the driving-point impedance of the aerial, the aerial current and the volts to ground at the driving point can be easily calculated.

Since the whole of the R.F. power supplied to the aerial is expended in the resistive component of the aerial impedance, it follows that,

$$I_A^2 R_s=P$$

and

$$\frac{V_A^2}{R_P}=P$$

where I_A=carrier current at driving point (amps R.M.S.),
R_s=equivalent *series* resistance (ohms),
V_A=carrier volts, R.M.S., to ground at driving point,
R_p=equivalent *parallel* resistance (ohms),
P=carrier power (watts).

Thus (see Fig. 55 (a))

$$I_A=\sqrt{\frac{P}{R_s}} \quad \ldots\ldots (31)$$

and (see Fig. 55 (b))

$$V_A=\sqrt{R_pP} \quad \ldots\ldots (32)$$

Example 22 : Calculate carrier current and volts to ground at the driving point of an aerial with an impedance of $Z_s=77-j292$ ohms, $Z_p=1185//-j312$ ohms, given that the carrier power is 10 kW.

Answer :

$$I_A=\sqrt{\frac{10000}{77}} = 11.4 \text{ amps., R.M.S.}$$

$$V_A=\sqrt{1185\times 10000}=3400 \text{ volts, R.M.S. (approx.)}$$

Example 23: Calculate the carrier currents and voltages, as marked in Fig. 56, given that the circuits are the alternatives of Example 19, page 28, and that the aerial carrier power is 50 kW. Assume component losses to be negligible.

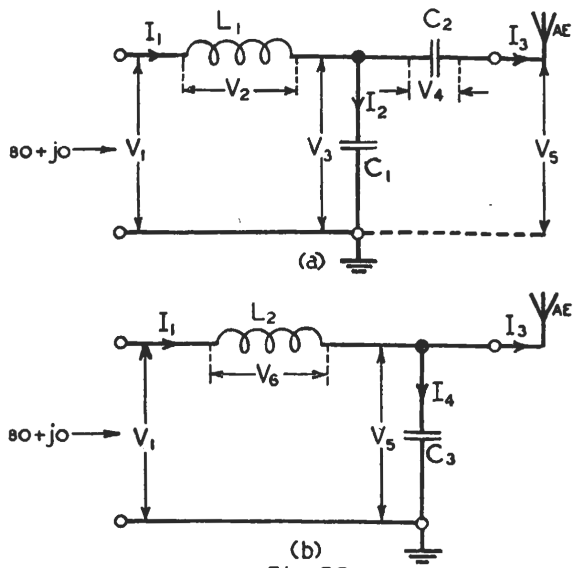


Fig 56

Answer: From the data of Example 19 and with reference to Fig. 56,

$$\text{Aerial } Z_s = 750 + j1120 \text{ ohms}$$
$$\text{,, } Z_p = 2422//j1623 \text{ ohms}$$
$$X_{L_1} = +231.5 \text{ ohms}$$
$$X_{C_1} = -259.1 \text{ ohms}$$
$$X_{C_2} = -1120 \text{ ohms}$$
$$X_{L_2} = +433 \text{ ohms}$$
$$X_{C_3} = -350.6 \text{ ohms}$$

Dealing first with the circuit illustrated in Fig. 56 (a). The current I_1 is the feeder current. As far as the feeder is concerned, the combination of aerial and matching network behaves simply as a resistance of 80 ohms in which a carrier power of 50 kW is expended.

Thus,

$$I_1 = \sqrt{\frac{50000}{80}} = 25 \text{ amps., R.M.S.}$$

$$V_1 = \sqrt{50000 \times 80} = 2000 \text{ volts, R.M.S.}$$

The voltage across the inductance L_1 is numerically equal to the product of the reactance of the coil and the current passing through the coil.

Thus,

$$V_2 = 231.5 \times 25 = 5800 \text{ volts, R.M.S. (approx.)}$$

Reference to the data and to Fig. 56 (a) will make it clear that, on the aerial side of the circuit, there is an impedance $750 + j(1120 - 1120) \doteq 750 + j0$ shunted across the condenser C_1.

Thus, C_1 is shunted by a resistance of 750 ohms.

In this resistance there is a power expenditure of 50 kW.

$$\therefore V_3 = \sqrt{50000 \times 750} = 6100 \text{ volts, R.M.S. (approx.)}$$

Dividing this voltage by the reactance of C_1 will give the value of I_2.

Thus,

$$I_2 = \frac{6100}{259.1} = 23.5 \text{ amps., R.M.S.}$$

The value of the current I_3 is the value of current at the aerial driving point. Since R_s of the series impedance of the aerial is 750Ω.

$$I_3 = \sqrt{\frac{50000}{750}} = 8.2 \text{ amps., R.M.S.}$$

The voltage V_4, across the condenser C_2, is numerically equal to the product of the reactance of C_2 and the current I_3.

$$\therefore V_4 = 1120 \times 8.2 = 9200 \text{ volts, R.M.S. (approx.)}$$

Since R_p of the parallel impedance of the aerial is 2422Ω and the power is 50 kW,

$$V_5 = \sqrt{50000 \times 2422} = 11000 \text{ volts, R.M.S. (approx.)}$$

Considering now the alternative circuit, Fig. 56 (b). The feeder current, feeder voltage and aerial driving-point current must necessarily be the same as for Fig. 56 (a). The voltage across the condenser C_3 is the voltage to ground at the aerial driving point and will be the same as V_5 calculated previously. Thus,

$$I_1 = 25 \text{ amps., R.M.S.}$$
$$V_1 = 2000 \text{ volts, R.M.S.}$$
$$I_3 = 8.2 \text{ amps., R.M.S.}$$
$$V_5 = 11000 \text{ volts, R.M.S. (approx.)}$$

The voltage across the inductance L_2 will be numerically equal to the product of the reactance and the current I_1. Thus,

$$V_6 = 433 \times 25 = 10800 \text{ volts, R.M.S. (approx.)}$$

Dividing V_5 by the reactance of C_3 will give the value of the current I_4. Thus,

$$I_4 = \frac{11000}{350.6} = 31.4 \text{ amps., R.M.S. (approx.)}$$

It is instructive to apply the principles of vector analysis to the current and voltage distribution in an A.T.H. network.

In illustration, this will now be done for the two cases dealt with in the above Example.

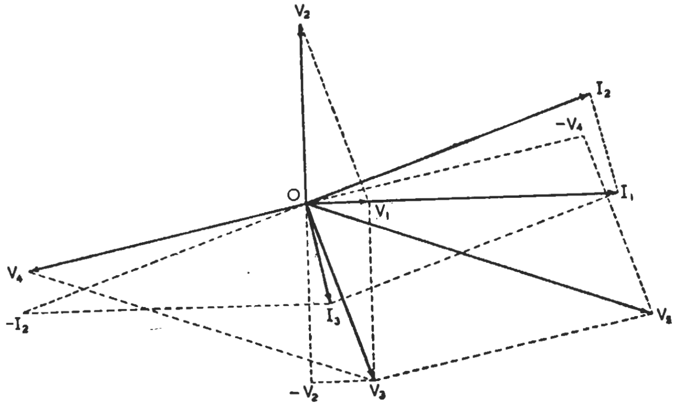


Fig 57

Fig. 57 is the vector diagram appropriate to Fig. 56 (a) and can be constructed as follows :—

Select convenient scales for the lengths of the voltage and current vectors.

Draw OI_1 of length representing to scale the feeder current of 25 amps.

Since the feeder is terminated by a non-reactive load, the feeder voltage, V_1, and current, I_1, will be in phase. Draw OV_1 of length representing to scale the feeder voltage of 2000.

Neglecting the resistance of the coil, L_1 (Fig. 56 (a)), it follows that the voltage, V_2, across the coil will be leading in phase by 90° on the current, I_1. Draw OV_2 90° from OI_1 (in an anti-clockwise sense) and of length representing to scale 5800 volts.

The voltage, V_3, across the condenser C_1 is equal to the vectorial difference between V_1 and V_2, and is obtained graphically by drawing the resultant of V_1 and $-V_2$, as follows : On V_2O produced through O mark off a length $O -V_2$ equal to OV_2. Complete the parallelogram, $O-V_2 V_3 V_1$, and draw the diagonal, OV_3. OV_3 is the vector which represents in magnitude and phase the voltage V_3.

The current I_2 is leading in phase by 90° on the voltage V_3. Draw OI_2 90° from OV_3 (in an anti-clockwise sense) and of length representing to scale 23.5 amps.

The current I_3 is equal to the vectorial difference between I_1 and I_2. On I_2O, produced through O, mark off a length $O -I_2$ equal to OI_2. Complete the parallelogram $O \; -I_2 \; I_3 \; I_1$ and draw the diagonal OI_3. OI_3 is the vector which represents in magnitude and phase the current I_3.

The voltage, V_4, across the condenser C_2 is lagging in phase by 90° on the current I_3. Draw OV_4 90° from OI_3 (in a clockwise sense) and of length representing to scale 9200 volts.

The voltage, V_5, from aerial driving point to ground is equal to the vectorial difference between V_3 and V_4. On V_4O, produced through O, mark off a length, $O \; -V_4$ equal to OV_4. Complete the parallelogram $O V_3 V_5 -V_4$ and draw the diagonal, OV_5. OV_5 is the vector which represents in magnitude and phase the voltage V_5.

In Fig. 57 certain dotted construction lines have been drawn additional to those detailed above and for the purpose of demonstrating that,

(*a*) I_1 is the resultant of I_2 and I_3,
(*b*) V_1 is the resultant of V_2 and V_3,
(*c*) V_3 is the resultant of V_4 and V_5.

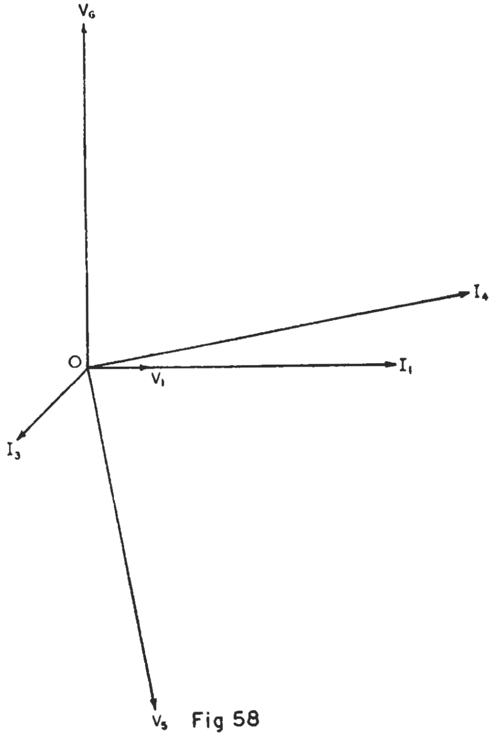


Fig 58

Working upon similar principles, the vector diagram appropriate to Fig. 56 (b) can be drawn.

This diagram, construction lines being omitted, is given in Fig. 58.

Example 24 : Design an A.T.H. set-up to meet the conditions specified below :—

(i) The transmitter operates on alternative frequencies of 668 kc/s, 1013 kc/s and 1474 kc/s at an aerial carrier power of 10 kW.

(ii) The aerial driving-point impedances at these frequencies are :—

668 kc/s	$Z_s = 12.5 - j16$	ohms
1013 ,,	$Z_s = 150 + j195$	,,
1474 ,,	$Z_s = 242 - j240$	,,

(iii) The feeder is of the concentric type with a characteristic impedance of 80 ohms.

(iv) The only condenser available is a semi-adjustable condenser, a schematic of which is illustrated in Fig. 59.

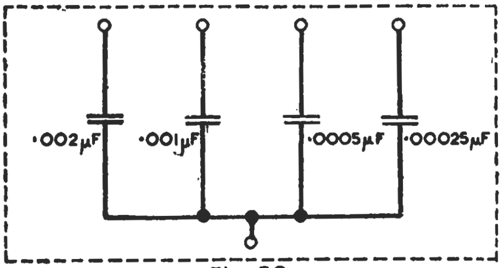


Fig 59

(v) This condenser is rated at 3000 V. R.M.S. at 500/1500 kc/s+100 per cent. tone modulation.

(vi) Two inductance coils are available, each having a maximum inductance of 50 μH and a current rating of 60 amps., R.M.S.

Answer :

Since the A.T.H. circuit has to be arranged for change-over between set-ups for three alternative frequencies, it is advisable at the outset to consider the possibilities where simplicity of change-over is concerned. In this respect it will be an advantage if the same basic form can be used for the set-up on each frequency.

Examining the equivalent series resistance values of the aerial shows that, whereas upwards transformation from R_s to R_o is required on 668 kc/s, downwards transformation is required on 1013 kc/s and 1474 kc/s.

As far as the equivalent parallel resistance values are concerned, a similar situation applies in respect of transformation from R_p to R_o.

With the object of achieving similarity of form in the set-ups, the use of a T-type network will be considered. Examination of the values of equivalent series aerial impedance makes it apparent that suitable set-up designs can be based on the series aspect of the aerial impedance.

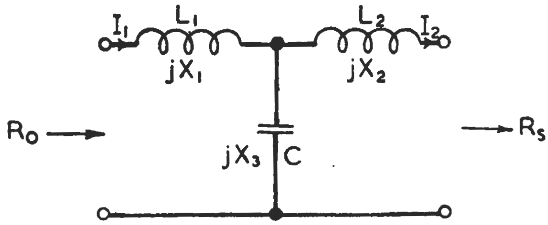


Fig 60

The circuit illustrated in Fig. 60 will, therefore, be adopted as the basic form of network unless investigation proves it to be unsuitable at any of the given frequencies.

It is possible at this stage to decide whether the specified coils will be satisfactory as regards current rating.

The current in L_1 is equal to the feeder current which in each case will be,

$$I_1 = \sqrt{\frac{10000}{80}} = 11.2 \text{ amps., R.M.S.}$$

The current in L_2 is equal to the current at the aerial driving point. This value will differ in the three cases and can be calculated from the values of equivalent series aerial resistance.

$$I_2 \text{ at } 668 \text{ kc/s} = \sqrt{\frac{10000}{12.5}} = 28.3 \text{ amps., R.M.S.}$$

$$I_2 \text{ ,, } 1013 \text{ kc/s} = \sqrt{\frac{10000}{150}} = 8.2 \quad \text{,,} \quad \text{,,}$$

$$I_2 \text{ ,, } 1474 \text{ kc/s} = \sqrt{\frac{10000}{242}} = 6.4 \quad \text{,,} \quad \text{,,}$$

These current values indicate that the coils will be run well within current rating even under modulated carrier conditions.

Since the maximum permissible voltage across the condenser is 3000 volts, the maximum permissible value of mid-shunt resistance is,

$$\frac{3000^2}{10000} = 900 \text{ ohms}$$

The minimum permissible numerical value of X_3 is equal to $\sqrt{R_s R_o}$. (See page 21.) Thus,

At 668kc/s, X_3 minimum $= \sqrt{12.5 \times 80} = 31.6$ ohms

,, 1013 ,, ,, ,, $= \sqrt{150 \times 80} = 109.5$,,

,, 1474 ,, ,, ,, $= \sqrt{242 \times 80} = 139.1$,,

From the above the maximum permissible capacity of C can be calculated.

At 668 kc/s,

$$C \text{ maximum} = \frac{10^3}{2\pi \times 668 \times 31.6} = .0075 \ \mu F \ \text{(app.)}$$

At 1013 kc/s,

$$C \text{ maximum} = \frac{10^3}{2\pi \times 1013 \times 109.5} = .0014 \ \mu F \ \text{(app.)}$$

At 1474 kc/s,

$$C \text{ maximum} = \frac{10^3}{2\pi \times 1474 \times 139.1} = .00078 \ \mu F \ \text{(app.)}$$

The maximum capacity to which the condenser illustrated in Fig. 59 can be adjusted is .00375 μF.

The values of .00375 μF, .001 μF and .00075 μF will therefore be investigated as regards suitability for the frequencies of 668 kc/s, 1013 kc/s and 1474 kc/s respectively. It will be necessary to make certain that (*a*) the inductance required in each coil does not exceed 50 μH and (*b*) the mid-shunt resistance does not exceed 900 ohms.

668 kc/s.

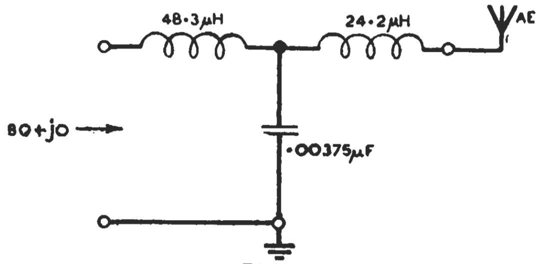


Fig 61

The reactance of .00375 μF is,

$$-\frac{10^9}{2\pi \times 668 \times 3750} = -63.5 \text{ ohms}$$

From equation (27), page 20,

$$X_1 = 63.5 \left[1 \pm \sqrt{\frac{80}{12.5} - \frac{80^2}{(-63.5)^2}} \right]$$

Taking the positive square root,

$$X_1 \text{ (Fig. 60)} = +202.8 \text{ ohms}$$

This value of reactance requires an inductance of

$$\frac{202.8 \times 10^3}{2\pi \times 668} = 48.3 \ \mu H$$

From equation (28),

$$X_2 = 63.5 \left[1 \pm \sqrt{\frac{12.5}{80} - \frac{12.5^2}{(-63.5)^2}} \right]$$

Taking the positive square root,

$$X_2 \text{ (Fig. 60)} = +85.3 \text{ ohms}$$

To this reactance must be added +16, required to balance out the aerial reactance of −16. Thus, the total reactance= +101.3 ohms.

This reactance requires an inductance of

$$\frac{101.3 \times 10^3}{2\pi \times 668} = 24.2 \ \mu H$$

It will be seen from the above that the specified coils will meet the inductance requirements. All that remains now as far as the 668 kc/s set-up is concerned is to make sure that the mid-shunt resistance does not exceed 900 ohms.

The mid-shunt resistance is equal to the equivalent parallel resistance of 12.5+j85.3 ohms

$$= 12.5\left(1 + \frac{85.3^2}{12.5^2}\right) = 595.5 \text{ ohms}$$

The set-up for 668 kc/s is illustrated in Fig. 61.

1013 kc/s.

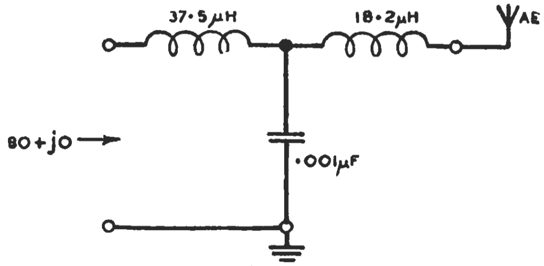


Fig 62

Working on the same lines as in the case of 668 kc/s, the following results are obtained for 1013 kc/s :—

Reactance of .001 μF at 1013 kc/s= −157 ohms.

$$X_1 \text{ (Fig. 60)} = +239.1 \text{ ohms}$$

$$\therefore L_1 = 37.5 \ \mu H.$$

$$X_2 \text{ (Fig. 60)} = +311.1 \text{ ohms.}$$

The series reactance of the aerial is +195, so the additional series reactance required is 311.1 − 195= +116.1 ohms.

This reactance is provided by an inductance of 18.2 μH.

The mid-shunt resistance, which is equal to the equivalent parallel resistance of 150+j311.1 ohms,

$$= 794.9 \text{ ohms}$$

The set-up illustrated in Fig. 62 will thus be satisfactory for 1013 kc/s.

1474 kc/s.

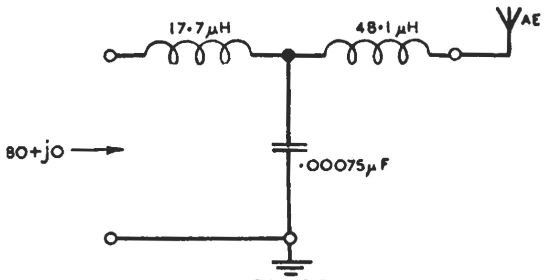


Fig 63

Reactance of .00075 μF at 1474 kc/s $= -143.7$ ohms.

X_1 (Fig. 60) $= +164.4$ ohms

$\therefore L_1 = 17.7\ \mu$H.

X_2 (Fig. 60) $= +206.2$ ohms

An additional series reactance of $+240$ is required to balance out the series aerial reactance of -240.

$$206.2 + 240 = +446.2 \text{ ohms}$$

This reactance is provided by an inductance of 48.1 μH.

The mid-shunt resistance $=418$ ohms.

The set-up for 1474 kc/s is illustrated in Fig. 63.

Matching Networks between Aerial and Balanced Feeder.

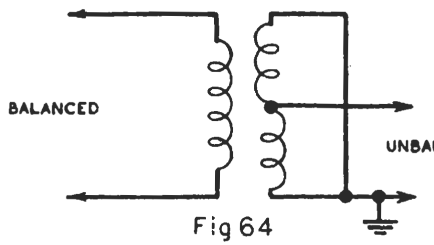


Fig 64

When a medium-wave aerial forming a circuit unbalanced with respect to earth is to be matched to a balanced feeder, the transition from unbalanced to balanced circuit conditions can be made by means of an R.F. transformer of the type illustrated schematically in Fig. 64.

The secondary consists of two similar coils wound in opposite sense and connected in parallel in such manner that the voltages produced by mutual induction from the primary are in phase. Inspection of Fig. 64 will make it clear that capacity between primary and earth, via the secondary coils, is fairly symmetrically distributed and will not seriously unbalance the primary circuit.

When the coupling is variable by movement of the secondary coils the movement of the two coils should be symmetrical with reference to the primary. For example, in the case where the primary is wound over the two secondaries the latter are mounted coaxially and move simultaneously either towards or away from each other.

Mutual Inductance Coupling.

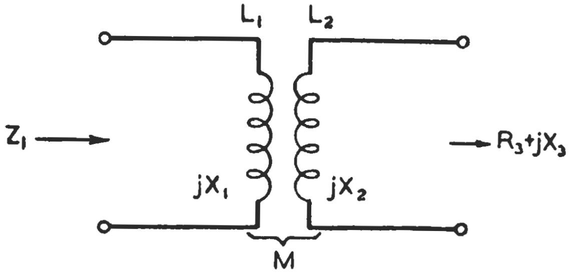


Fig 65

As explained in standard text-books, the coupling of a loaded secondary coil to a primary coil modifies the effective impedance of the primary circuit. The effect is equivalent to the insertion in series with the primary of an impedance Z, such that,

$$Z = \frac{(\omega M)^2}{Z_2} \qquad \text{.......................................(33)}$$

where $\omega = 2\pi \times$ frequency.

M = mutual inductance.

Z_2 = total equivalent series impedance of the secondary circuit.

If $Z_2 = R_s + jX_s$

$$Z = \frac{(\omega M)^2}{R_s + jX_s}$$

$$= \frac{(\omega M)^2 (R_s - jX_s)}{R_s^2 + X_s^2}$$

$$= \frac{(\omega M)^2 R_s}{R_s^2 + X_s^2} - j\left[\frac{(\omega M)^2 X_s}{R_s^2 + X_s^2}\right] \qquad \text{...........(34)}$$

It will be observed from equation (34) that the reactance "reflected" into the primary by the coupled secondary is *opposite* in sign to the total series reactance of the secondary circuit.

In the case illustrated in Fig. 65, and assuming that the resistances of the primary and secondary coils are sufficiently small to be neglected,

$$X_s = (X_2 + X_3)$$

$$R_s = R_3$$

$$Z_1 = \frac{(\omega M)^2 R_3}{R_3^2 + (X_2 + X_3)^2} + j\left[X_1 - \frac{(\omega M)^2 (X_2 + X_3)}{R_3^2 + (X_2 + X_3)^2}\right] \qquad (35)$$

If the secondary circuit is adjusted to unity power factor, i.e. if X_2 and X_3 are equal and opposite in value,

$$Z_1 = \frac{(\omega M)^2}{R_3} + jX_1 \qquad \text{..............................(36)}$$

Matching Networks.

The conditions expressed by equation (36) are those which normally apply in the case of an R.F. transformer employed in a medium-wave A.T.H. network.

It is to be particularly noted that R_3 is effectively in series with the parallelled secondary coils and $(\omega M)^2/R_3$ in series with the primary coil. Thus,

$$I_2=\sqrt{\frac{P}{R_3}} \quad \text{.....(37)}$$

and

$$I_1=\frac{\sqrt{PR_3}}{\omega M} \quad \text{.....(38)}$$

where I_2=total secondary current (amps., R.M.S.).
P=power (watts).
R_3=effective resistance in series with parallelled secondaries (ohms).
I_1=primary current (amps., R.M.S.).
M=mutual inductance (H).
$\omega=2\pi\times$frequency (c/s).

The current in each secondary coil $=I_2/2$.

The voltage across each secondary coil is

$$V_2=\sqrt{PR_4} \quad \text{.....(39)}$$

where R_4=equivalent parallel resistance of R_3+jX_2.
X_2=reactance of the two secondary coils in parallel.

The voltage across the primary is

$$V_1=\sqrt{PR_5} \quad \text{.....(40)}$$

where R_5=equivalent parallel resistance of $\frac{(\omega M)^2}{R_3}+jX_1$
X_1=reactance of primary coil.

Example 25: In the case of an R.F. transformer of the type illustrated in Fig. 64, calculate the value of ωM (mutual reactance), given that the current in each secondary coil=12 amps., the primary current=40 amps., and the power=50 kW. Assume that the secondary circuit is adjusted to unity power factor.

Answer:

The total secondary current=24 amps.

R_3 (resistance in series with secondaries)=

$$\frac{50000}{24^2}=86.8 \text{ ohms.}$$

$\frac{(\omega M)^2}{R_3}$ (resistance in series with primary)=

$$\frac{50000}{40^2}=31.2 \text{ ohms.}$$

$$\therefore \frac{(\omega M)^2}{86.8}=31.2 \text{ ohms}$$

$$\therefore \omega M=\sqrt{86.8\times 31.2}=+52 \text{ ohms}$$

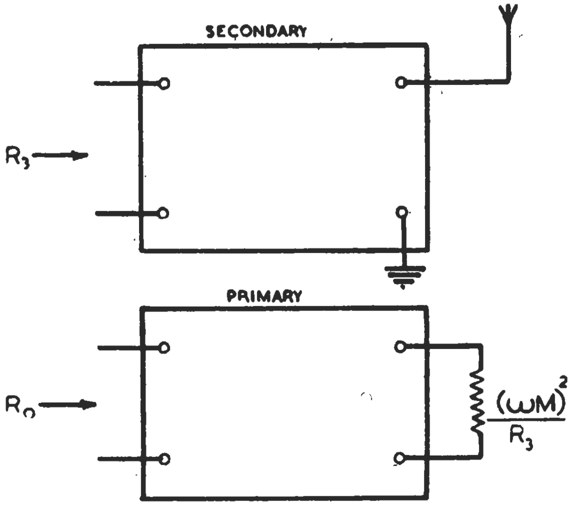


Fig 66

To understand the setting up of an A.T.H. network involving a transformer, it is useful to regard the complete network between aerial and feeder as consisting of two distinct sections (Fig. 66).

(*a*) A network, *including the parallelled secondary coils of the transformer*, which matches the aerial impedance to R_3.

(*b*) A network, *including the primary coil of the transformer*, which matches $(\omega M)^2/R_3$ to the characteristic impedance of the feeder.

It will be seen that, provided the required values of R_3 and $(\omega M)^2/R_3$ are known, the network design can be carried out on the lines already described.

It must be noted that, although L_2 and L_1 (Fig. 65) can be treated as parts of transforming networks, there is not the freedom for variation of inductance that has been assumed for the coils in the matching networks that have so far been discussed in detail in this Instruction. The transformation from R_3 to $(\omega M)^2/R_3$ is dependent upon the mutual inductance between L_2 and L_1, and neither of these coils must be adjusted to such value that it is impossible by normal coupling adjustment to secure the requisite value of M.

It is usual in A.T.H. circuits to use simple L networks to transform from $(\omega M)^2/R_3$ to R_o, and it may well happen that the reactance of the transformer primary is of unsuitable value for direct use as the series element of such a network. If the value of series reactance required is not satisfactory as a value of reactance for the transformer primary itself, it will be necessary to compensate by the addition of reactance external to the primary (assuming that the secondary circuit is adjusted to unity power factor).

Any series reactance added to the primary circuit must be so connected that it does not upset the balance of the latter with respect to earth. If the reactance of the primary is too large for the matching network the use of two condensers (one on each side of the primary) can be avoided if the primary is split at the centre. In this case one condenser can be used, as illustrated in Fig. 67.

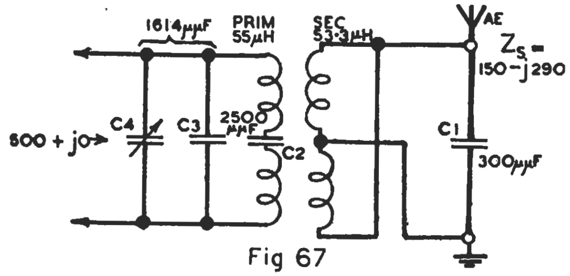


Fig 67

Fig. 67 illustrates an A.T.H. set-up for 668 kc/s (feeder R_o=500 ohms) in which the transformed resistance in series with the secondary coils is 79 ohms and the coupling is adjusted to refer 40 ohms to the primary. The network on the secondary side is based upon the parallel aspect of the aerial impedance, which is 711//−j368 ohms.

The reactance of the condenser $C_1 = -793.4$.

The combined reactance of the condenser C_1 and the equivalent parallel reactance of the aerial,

$$= \frac{(-368) \times (-793.4)}{-368-793.4} = -251.4 \text{ ohms}$$

The combination of aerial impedance and C_1 is thus 711//−j251.4 ohms, the equivalent series impedance being 79−j223.6 ohms.

By adjusting the total reactance of the secondary coils to +223.6(53.3μH) a transformed resistance value of 79 ohms is obtained.

Assuming that the primary inductance is 55μH and that the coupling is adjusted until 40 ohms is referred from secondary to primary, the series impedance of primary and referred resistance =40+j230.9 ohms.

The reactance of the condenser $C_2 = -95.2$.

The impedance between the primary terminals, with C_2 connected=40+j135.7 ohms, the parallel equivalent of which=500//j147.4 ohms.

The parallel reactance is balanced out by a reactance of −147.4 ohms formed by the parallelled condensers C_3 and C_4, the joint capacity of which is 1614 μμF.

The Practical Setting-up of A.T.H. Circuits.

The first step must, naturally, be the measurement of the impedance that it is required to match to the feeder characteristic impedance. So far in this Instruction this has been regarded as the impedance at the aerial driving point. In cases, however, where the system includes a rejector connected between the aerial and the matching network it is most important to make a bridge measurement with the rejector in position in series with the aerial and to base the matching design upon the latter measurement. Given the L and C values of the rejector and the impedance at the driving point of the aerial, it is a simple matter to calculate the theoretical impedance with the rejector in circuit, but in practice it will frequently be found that the actual impedance differs considerably from the calculated value. This is due to the effect of the capacity between the rejector components and earth. Such an effect can be very pronounced when the rejector condenser is of large physical size.

After the circuit design has been worked out and it comes to a matter of setting it up, the exact procedure must necessarily depend upon the nature of the circuit, and two representative cases are described below.

It should be noted that the effect of strays should always be regarded as a possibility, and where the set-up adjustments seem to be departing from expected values such a possibility should be given first consideration.

Where this occurs it must necessarily modify some of the adjustments. As to whether or not it necessitates a complete re-design is simply the issue as to whether the feeder can, or cannot, be matched, using the components available.

The first case to be considered will be that illustrated in Fig. 50 (b), page 28 (Example 20). Assuming that both the coils are variable, the first step should be to adjust the aerial coil.

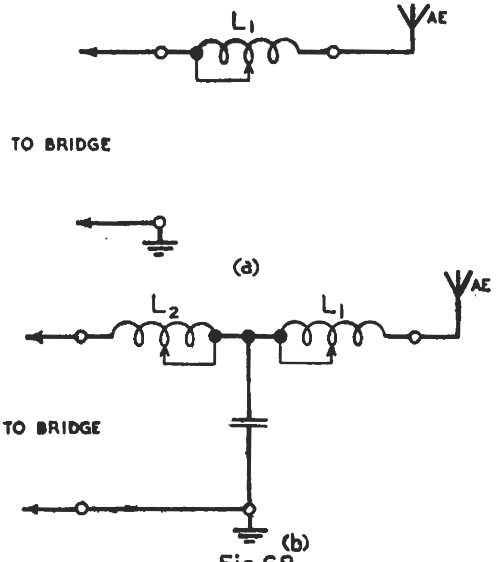


Fig 68

The purpose of this coil is to balance out the equivalent series reactance of the aerial. With the bridge connected as shown in Fig. 68 (a), L_1 should be adjusted until unity power factor is indicated. The other coil functions as the series element of a downwards L transforming network and with the bridge connected as shown in Fig. 68 (b), L_2 should be adjusted until unity power factor is indicated. At the same time the transformed resistance value should be checked to ensure that it is satisfactory for matching the feeder.

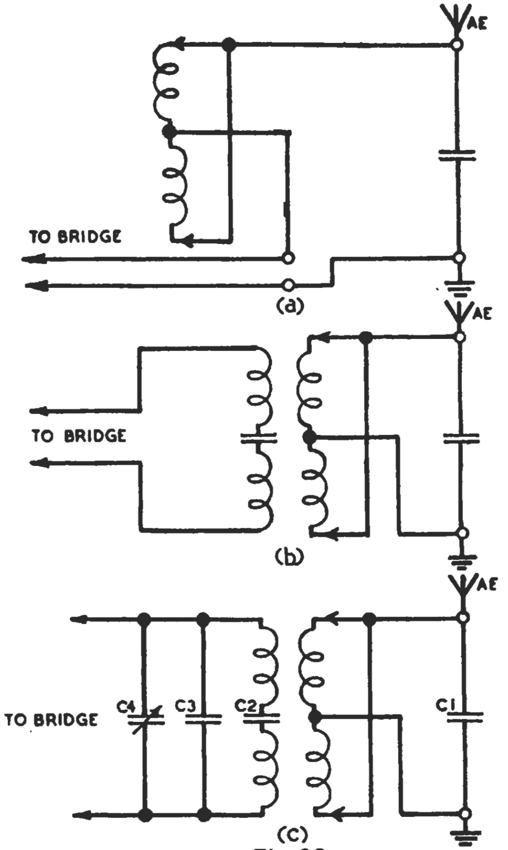


Fig 69

The second case to be considered is that of the circuit illustrated in Fig. 67.

The first step should be to open the primary circuit, connect the bridge between the parallelled secondary coils and earth, as shown in Fig. 69 (a), and adjust the secondary inductance until unity power factor is obtained. Equal adjustments should be made to each of the secondary coils. The transformed resistance value should be checked to ensure that it is at or very near to the required value.

Next the bridge should be connected to the primary as shown in Fig. 69 (b) and, with the secondary network complete, the coupling should be adjusted until the series resistive component of the measured impedance has the required value.

If the method of varying coupling is such as to affect the total inductance of the parallelled secondaries and if the amount of coupling adjustment is considerable it would be advisable to repeat the first step, described above.

Finally, the bridge should be connected as shown in Fig. 69 (c) and C_4 adjusted until zero phase angle is obtained, the transformed resistance value being checked to see that it is satisfactory. It should be noted that if C_2 is adjustable it is this component which should be changed in value if the transformed resistance presented to the feeder is excessively high or low.

SECTION J. MATCHING BETWEEN TERMINATED FEEDER AND TRANSMITTER.

If the feeder is correctly terminated at the aerial end it will represent at the transmitter end a load equal to the characteristic impedance of the feeder.

The output valve or valves of the transmitter must work into an anode load resistance of a value dependent upon the design of the output stage and some form of matching network is necessary in order to match the feeder load to the anode load.

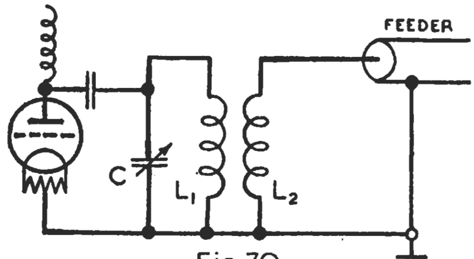


Fig 70

Fig. 70 illustrates a very simple form of circuit typical of a small transmitter. L_1 and C comprise the tuned output circuit (" tank " circuit) of the final valve stage. L_2 is variably coupled to L_1.

The functioning of this circuit can be explained on the lines taken in describing matching networks in the previous sections of this Instruction. L_2 and L_1 constitute an R.F. transformer. Resistance and reactance are referred from the secondary to the primary (see equation 34, page 36). The reactance of L_1 (added to any reactance referred from the secondary), and the reactance of C, provide an L network transforming the series resistance referred from the secondary up to a value which constitutes the anode load resistance.

Control on the value of this anode load resistance can be exercised by variation of coupling between L_2 and L_1, upon which the resistance referred to primary depends. It will be understood that in the case of the circuit of Fig. 70 any change of coupling will change the effective reactance in the primary, necessitating compensation by adjustment of the tank condenser C. The extent of this effect will be dependent upon the reactance and resistance of the secondary circuit and can, of course, be obviated if the secondary circuit is adjusted to unity power factor.

In cases where the tank circuit has the simple character of that illustrated in Fig. 70 and where the R.F. transformer is designed so that its primary can act as the series element of an L network, the tank circuit should be tuned with the secondary circuit disconnected. Such tuning is usually done on power, rather than by bridge measurements.

With the secondary circuit complete and normally coupled to the primary, the primary should be retuned. It is essential that any reactance referred from secondary to primary shall not be so large that the tuning of the tank circuit cannot be " picked up " on the variable tuning element.

Hard and fast rules cannot be laid down as to the maximum value of referred reactance that can be tolerated in the primary circuit. It is to be noted, however, that in those cases where the variable tuning element of the tank circuit exercises only very small control of reactance, very little referred reactance can be permitted. It should be noted, also, that, whereas tuning of the tank circuit by shunt condenser does not vary the parallel resistance component constituting the load resistance of the valve, tuning by series inductance does do so. Thus considerations of load resistance have to be allowed for in cases where series inductance tuning is employed.

It is not always possible, particularly in high power installations, to design the output transformer so that its primary circuit functions in the simple manner described above.

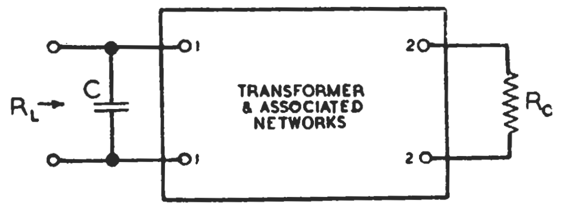


Fig 71

In such cases the circuits will be understood if a viewpoint based upon the schematic of Fig. 71 is adopted.

In Fig. 71 let R_L be the required value of anode load resistance and C the capacity (e.g. tuning condenser, valve capacity, etc.) connected across it. The transformer is then assumed to be associated with such reactive networks that when the terminals 2, 2 are terminated by the characteristic impedance of the feeder, terminals 1, 1 shall present an impedance equal to R_L in parallel with a positive reactance numerically equal to X_C.

On page 36 the subject of the influence of secondary circuit impedance upon the primary impedance of an R.F. transformer was introduced. In the case of the simple circuit of Fig. 70 any reactance reflected from secondary to primary was regarded as incidental. In more complicated circuits the matter may be quite different and reactance may be deliberately referred from secondary to primary in order to adjust the primary impedance to a particular value.

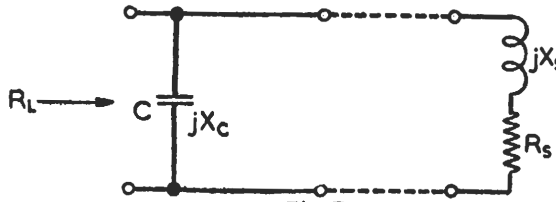


Fig 72

In working out a matching circuit design involving a given R.F. transformer the problem, once the effective impedance between the primary terminals has been determined, becomes that of transforming this impedance to an impedance represented by the required anode load resistance shunted by a positive reactance equal in magnitude to the reactance of the capacity across the valve.

In Fig. 72 R_s+jX_s represents the equivalent series impedance between the transformer primary terminals. R_L is the required value of anode load resistance and C is the anode-cathode capacity.

If R_s and X_s can be given such values that their equivalent parallel resistance is equal to R_L and their equivalent parallel reactance is equal in value (but opposite in sign) to X_c, then an impedance match will be secured by direct connection as shown in Fig. 72.

Expressing the same fact in terms of equivalent series impedances, if the equivalent series resistance of $R_L//jX_c$ is equal to R_s and its equivalent series reactance is equal in value (but opposite in sign) to jX_s, then the direct connection will provide the impedance match.

Assuming that the impedance appearing at the primary terminals of the transformer is unsuitable for direct connection as in Fig. 72, the addition of series and shunt reactances will generally be required, as indicated in Fig. 73.

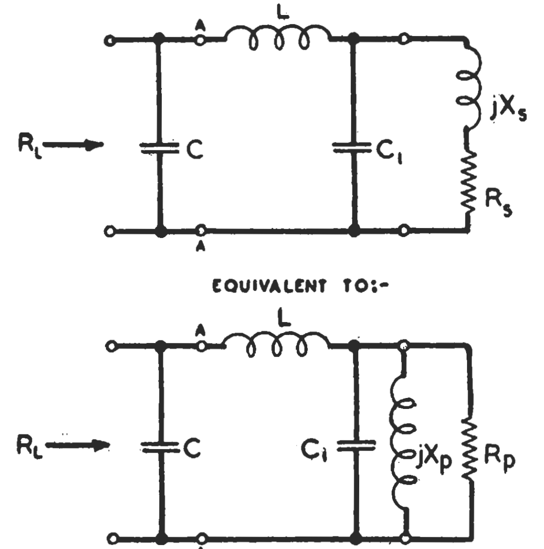


Fig 73

The object of the added components L and C_1 is to transform the impedance at the primary terminals of the transformer (R_s+jX_s or $R_p//jX_p$) so as to present at the point A.A. of the network a new impedance $R_{s1}+jX_{s1}$, whose parallel equivalent resistance shall be equal to R_L and whose parallel equivalent reactance shall be equal to $-X_c$, as already explained.

Considering now the effect of the addition of the condenser C_1 in parallel with the primary winding of the transformer, X_{c1} and X_p, in parallel are required to form a total parallel reactance, X_{p1} sufficient to transform R_p to the desired value of equivalent series resistance, R_{s1}.

$$X_{p1}=\frac{X_pX_{c1}}{X_p+X_{c1}}$$

due attention being paid to signs of X_p and X_{c1}.

As the sign of X_{p1} has not been specified, two values of X_{c1} will satisfy this condition. If X_{c1} is negative and greater than X_p, X_{p1} is positive. This method of adjustment is sometimes called " undertuning " the transformer. If, on the other hand, X_{c1} is negative and less than X_p, X_{p1} will be negative in sign, and this method is sometimes called " over-tuning " the transformer.

The appropriate value of R_{s1} having been achieved, X_L is now specified to give the required value of X_{s1}. The value of X_L will, of course, be different in the two cases of " undertuning " and " overtuning " respectively.

Example 26 : Calculate the values of the reactive elements and draw the circuits to match a transformer primary effective impedance of 29+j180.3 ohms so as to provide an anode load resistance of 2000 ohms, the total capacity between anode and cathode having a reactance of −400 ohms. Deal with two cases : (*a*) using a primary shunt condenser for " undertuning," (*b*) using a primary shunt condenser for " overtuning."

Answer (*a*) :

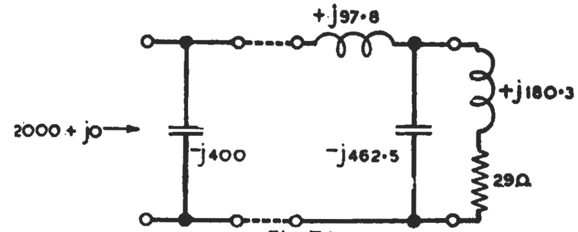


Fig 74

The impedance to which the transformer primary impedance is to be converted by the network is 2000//j400 ohms, the series equivalent of which is 76.9+j384.7 ohms.

What is required of the condenser shunted across the primary is that it shall raise the primary parallel reactance to such a value that the equivalent series resistance is 76.9 ohms.

The equivalent parallel impedance of 29+j180.3 ohms is 1150//j184.7 ohms.

$$\text{Let } m=\frac{1150}{76.9}=14.96$$

$$\sqrt{m-1}=3.74.$$

The parallel reactance required is,

$$\pm\frac{m\times 76.9}{\sqrt{m-1}}=\pm\frac{14.96\times 76.9}{3.74}$$

$$=\pm\ 307.5 \text{ ohms}$$

The positive value of reactance can be produced if X_c is given the value obtained by solving the equation,

$$\frac{184.7X_c}{184.7+X_c}=+307.5$$

whence,

$$X_c=-462.5 \text{ ohms}$$

The condenser and the transformer primary impedance provide a joint impedance of 1150// +j307.5 ohms, the equivalent series impedance being 76.9+j286.9 ohms.

The impedance required is 76.9+j384.7 ohms, which can now be obtained by the addition of a series reactance of 384.7−286.9= +97.8 ohms.

The circuit is illustrated in Fig. 74.

Answer (*b*):

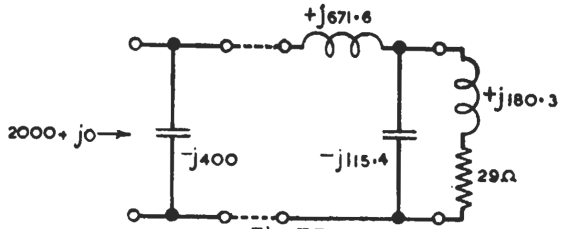


Fig 75

The negative value of reactance can be produced if X_c is given the value obtained by solving the equation,

$$\frac{184.7X_c}{184.7+X_c} = -307.5$$

whence,

$$X_c = -115.4 \text{ ohms}$$

The condenser and the transformer primary impedance provide a joint impedance of 1150// −j307.5 ohms, the equivalent series impedance of which is 76.9−j286.9 ohms.

As before, the impedance required is 76.9 + j384.7 ohms, which can be obtained by the addition of a series reactance of 286.9+384.7= +671.6 ohms.

The circuit is illustrated in Fig. 75.

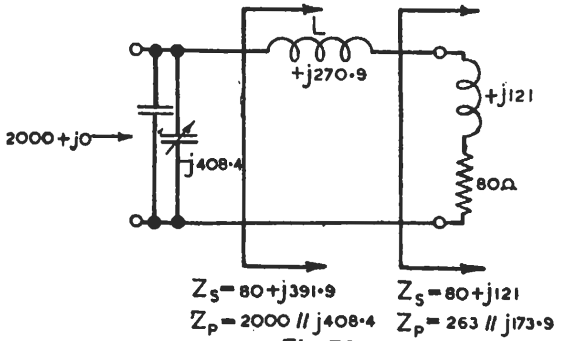


Fig 76

Fig 76 is an impedance schematic illustrating a case in which a transformer primary impedance of 80+j121 ohms is associated with a network to produce an anode load resistance of 2000 ohms. A variable condenser is connected between anode and cathode and the reactance of −408.4 ohms shown in Fig. 76 represents the combined reactance of this condenser and the valve capacity.

It should be noted that with this type of circuit control over the value of anode load resistance can be secured by adjustment of the inductance L. Adjustment for the condition of unity power factor is made by means of the variable condenser.

It should be understood, in connection with Figs. 74, 75 and 76 that, if the circuit must be balanced, any series connected reactive elements must be split between the two sides of the circuit.

Such would be the case when the final stage of the transmitter is of push-pull type.

In connection with the matter, previously mentioned, of adjusting the primary effective impedance to a particular value by means of reactance deliberately referred from the secondary circuit of the output transformer, the following should be noted :—

If a condenser is connected in series between the secondary winding of the transformer and the terminating resistance (i.e. the output feeder R_o), the equivalent parallel resistance of the primary will be a minimum and also independent of frequency if the condenser has the value,

$$C = \frac{L_1}{\omega^2 M^2}\left(\frac{K^2}{1-K^2}\right) = \frac{L_1}{\omega^2 (L_1 L_2 - M^2)} \mu\mu F \ldots (41)$$

where L_1=inductance of primary winding (μH).
L_2= ,, ,, secondary winding (μH).
$\omega = 2\pi \times$ frequency (kc/s).
M=mutual inductance (μH).

$$K = \text{coefficient of coupling} = \frac{M}{\sqrt{L_1 L_2}}$$

The value of the primary R_p will be

$$R_p \text{ (minimum)} = R_o\left(\frac{L_1}{M}\right)^2 \ldots\ldots (42)$$

where R_o=secondary load resistance and L_1 and M are defined as above.

The value of the primary X_p will be

$$X_p = \omega L_1 \ldots\ldots (43)$$

i.e. the reactance of the primary winding itself.

If a parallel connected secondary condenser is used, instead of a series connected condenser, the reactance, XC, of the condenser must be such that,

$$\frac{(\omega M)^2}{X_1} = X_2 + \frac{R_o^2 X_c}{R_o^2 + X_c^2} \ldots\ldots (44)$$

where X_1=reactance of primary winding
X_2= ,, ,, secondary ,,
$\omega = 2\pi \times$ frequency, (c/s)
M=mutual inductance (H)
R_c=secondary load resistance.

If the transformer has a 1 : 1 ratio (i.e. primary and secondary voltages and currents equal), and if a parallel connected secondary condenser is employed to produce minimum R_p in the primary, the following relationships exist :—

$$X_1 = X_2 = -X_c = R_o\sqrt{\frac{r_s}{R_o - r_s}} \ldots\ldots (45)$$

$$\omega M = r_s\sqrt{\frac{R_o}{R_c - r_s}} \ldots\ldots (46)$$

$$K = \sqrt{\frac{r_s}{R_o}} \ldots\ldots (47)$$

where r_s=equivalent series resistance of secondary circuit,

and the other symbols are defined as above.

The principles underlying equations (41) to (47) are of limited application, but examples will be found in certain high power installations.

SECTION K. REJECTORS.

The operation of two or more M.W. transmitters on the same station site leads to problems associated with the presence in one aerial circuit of power radiated from the other. Such transference of power tends to be greater the smaller the difference between the two carrier frequencies.

In the cases where the power in an aerial circuit, due to a neighbouring radiator, would be excessive if no preventative measures were taken, it is the usual practice to insert a rejector in the aerial circuit. This rejector would be tuned to present maximum impedance at the unwanted frequency, i.e. the frequency of the neighbouring radiator.

A less common application is that in which rejectors are employed for the purpose of enabling two independent transmitters to radiate from the same aerial system.

A third application is that of harmonic attenuation, rejectors being employed for this purpose in certain cases.

Some general considerations relating to rejectors are dealt with in this section, but it must be understood that the design of a rejector to satisfy the requirements of any particular case is largely a matter of considerable practical experience.

It is desirable that a rejector shall have a high Q value in order that the impedance shall be very high at the unwanted frequency but comparatively low at the wanted frequency.

Normally the tuning adjustment of the rejector, which is a highly critical adjustment, is carried out by varying the inductance, the capacity being fixed.

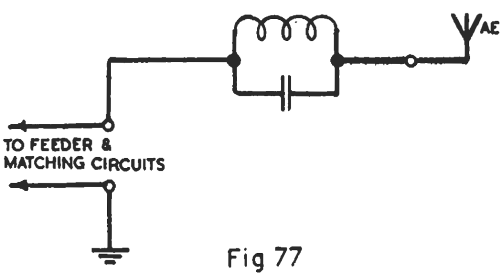


Fig 77

Consider the case of a rejector inserted in the aerial circuit of a transmitter to minimise currents due to a neighbouring radiator (Fig. 77).

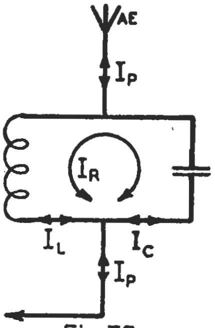


Fig 78

If X_L and X_C are the reactances of the inductance and capacity, respectively, *at the pass frequency*, the equivalent series reactance represented by the rejector at that frequency is,

$$X \text{ pass} = \frac{X_L X_C}{X_L + X_C} \quad \text{due regard being paid to signs} \ldots\ldots (48)$$

From equation (48) it will be seen that if $X_L > -X_C$, at the pass frequency, the rejector is equivalent to a series-connected capacity, since X pass is then negative. This will be the case if the reject frequency is lower than the pass frequency. If $X_L < -X_C$, at the pass frequency, the rejector is equivalent to a series-connected inductance. This condition occurs if the reject frequency is higher than the pass frequency.

In terms of the pass and reject frequencies and the capacity of the condenser,

$$X \text{ pass} = -\frac{10^9 F_P}{2\pi C\,(F_P + F_r)\,(F_P - F_r)} \ldots\ldots (49)$$

where F_P=pass frequency (kc/s)

F_r=reject frequency (kc/s)
C=capacity ($\mu\mu$F)

The derivation of the above formula is given in Appendix A, page 50.

In calculating the ratings required for the components of the rejector approximate calculations of current values can be made by neglecting component losses. In Fig. 78 I_P represents the pass current, i.e. the current passed through the rejector by the transmitter to which the aerial is connected. I_r represents the reject current, i.e the circulating current set up in the rejector at the reject frequency. It must be realised that this circulating current may be many times greater than I_P.

I_P will be considered first. If the rejector is connected between the aerial driving point and the feeder matching network, as illustrated in Fig. 77, then I_P is the normal driving point current, the calculation of which has already been described (equation (31), page 31).

Inside the rejector the pass current divides into two components I_l in the inductance branch and I_c in the condenser branch. The pass current component in the condenser branch is,

$$I_c = I_P\left[\frac{1}{1-\left(\frac{F_r}{F_P}\right)^2}\right] \quad \text{..............(50)}$$

where I_P = total pass current
F_r = reject frequency
F_P = pass frequency

It will be observed that I_c is independent of the C/L ratio.

The derivation of the above formula is given in Appendix A, page 51.

The pass current component in the inductance branch is

$$I_l = I_P\left[\frac{1}{1-\left(\frac{F_P}{F_r}\right)^2}\right] \quad \text{....................(51)}$$

Thus I_l also is independent of the C/L ratio.

The derivation of the above formula is given in Appendix A, page 51.

It will be noted that I_l and I_P are always opposite in sign. This indicates that the current in the inductance is always 180° out of phase with that in the condenser, which would be expected as series resistance is negligible. Moreover, when $F_r > F_P$, I_c is negative and I_l positive. If, however, $F_P > F_r$, then I_c is positive and I_l negative. The significance of the negative sign is that the current to which it refers is 180° out of phase with I_P, the aerial current.

Since $I_P = I_l + I_c$
$I_l = I_P - I_c$

due regard being paid to signs.

Since I_c is negative when $F_r > F_P$ and positive when $F_P > F_r$, it follows that the current in the inductance is equal to the numerical sum of the currents in the condenser and in the aerial (as measured, for example, by a thermo-ammeter) in the former case, and to their numerical difference in the latter case.

Example 27 : A 668 kc/s rejector is connected below the aerial driving point of a transmitting installation radiating on 804 kc/s. The carrier current at the aerial driving point is 10 amps. R.M.S. Calculate the current, at the pass frequency, in the rejector condenser.

Answer: By equation (50),

$$I_c = 10\left[\frac{1}{1-\left(\frac{668}{804}\right)^2}\right]$$

$= 32.3$ amps.

Example 28 : A 668 kc/s rejector employs a condenser of 6300 $\mu\mu$F. What is the equivalent series reactance of the rejector at 804 kc/s?

Answer : By equation (49), the reactance at 804 kc/s

$$= -\frac{10^9 \times 804}{2\pi \times 6300\,(804+668)\,(804-668)}$$

$$= -101.4 \text{ ohms}$$

Consideration will now be given to I_r, the circulating current at the reject frequency. This will be dependent upon the voltage produced across the rejector by the unwanted transmission.

For design purposes it is frequently necessary to make an estimate of I_r before the choice of components can be made. Obviously the voltage produced across the rejector cannot be measured until the rejector has been installed and adjusted. What can be done, however, is to measure the voltage produced by the unwanted transmission between the aerial driving point and earth with the driving point disconnected on the A.T.H. side. The value of voltage so determined represents the voltage that would be produced across the rejector by the unwanted transmission if the impedance of the rejector were infinite at the reject frequency. In practice the effective resistance of the rejector will be finite and the voltage determined as above must be treated as a voltage value somewhat in excess of what will be obtained in practice. It is, however, a satisfactory value to use in determining safe ratings of the components.

$$I_r = \frac{V_r \omega_r C}{10^9} \text{ amps., R.M.S.} \quad \text{......................(52)}$$

where V_r = voltage, measured as described (volts, R.M.S.)
$\omega_r = 2\pi \times$ reject frequency (kc/s)
C = capacity ($\mu\mu$F)

The total current in the condenser consists of two components, I_c and I_r, as indicated in Fig. 78. Since these components are of different frequencies,

$$I_c \text{ (total)} = \sqrt{I_c^2 + I_r^2} \text{ amps., R.M.S.} \quad \text{.....(53)}$$

where I_c = component at pass frequency (amps., R.M.S.)
I_r = component at reject frequency (amps., R.M.S.)

Similarly,

$$I_l \text{ (total)} = \sqrt{I_l^2 + I_r^2} \quad \text{......................(54)}$$

where I_l = component at pass frequency (amps., R.M.S.)
I_r = component at reject frequency (amps., R.M.S.)

SECTION L. ACCEPTORS.

Fig 79

The fact that a series combination of L and C, Fig. 79, can be so proportioned that at a wanted frequency it is equivalent to a negative reactance while at an (higher) unwanted frequency it acts virtually as a short-circuit, provides useful possibilities where harmonic attenuation is concerned.

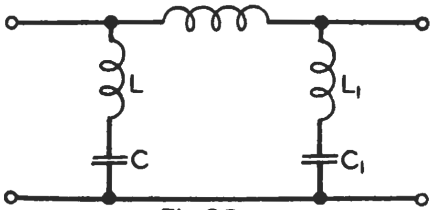


Fig 80

It becomes possible, for example, to use such " acceptor " combinations in place of normal shunt capacitive arms in networks as illustrated in Fig. 80. By suitable proportion of values, L C and L_1 C_1 will act as capacities of the correct value at the fundamental frequency but as virtual short-circuits at a chosen harmonic frequency.

If it be required that a series L C combination (Fig. 79) shall act as an acceptor at a frequency F_o and as a reactance X'_c at a frequency F, the values of L and C must be such that,

$$\omega_o L - \frac{1}{\omega_o C} = 0 \quad \text{.............................(55)}$$

$$\omega L - \frac{1}{\omega C} = X_c' \quad \text{...............................(56)}$$

where $\frac{\omega_o}{2\pi} = F_o$ (c/s)

$\frac{\omega}{2\pi} = F$ (c/s)

L=inductance (H)

C=capacity (F)

From equations (55) and (56), it can shown that

$$C = \frac{\omega^2 - \omega_o^2}{\omega_o^2 \, \omega X'_c} \text{ (F)} \quad \text{..........................(57)}$$

The derivation of the above formula is given in Appendix A, page 51.

By substituting the value of C in either equation (55) or (56) the value of L may be determined.

If F_o is the nth harmonic of F, then $\omega_o = n\omega$. Substitute $\omega_o = n\omega$ in equation (57).

$$\therefore C = \frac{1 - n^2}{n^2 \omega X'_c} \text{ (F)} \quad \text{............................(58)}$$

Example 29 : What are the L and C values for an acceptor to resonate at the 3rd harmonic of 668 kc/s and to be equivalent to a reactance of −952 ohms at the fundamental frequency ?

Answer :

By equation (58),

$$C = \frac{1 - 9}{-(9 \times 2\pi \times 668 \times 10^3 \times 952)} \text{ F}$$

$$= \frac{8 \times 10^9}{9 \times 2\pi \times 668 \times 952} \ \mu\mu\text{F}$$

$$= 222.5 \ \mu\mu\text{F}$$

From equation (55),

$$L = \frac{1}{\omega_o^2 C}$$

$$\omega_o = 3\omega$$

$$\therefore L = \frac{10^{12}}{(2\pi \times 3 \times 668 \times 10^3)^2 \times 222.5} \text{ H}$$

$$= \frac{10^{12}}{(2\pi \times 3 \times 668)^2 \times 222.5} \ \mu\text{H}$$

$$= 28.3 \ \mu\text{H}$$

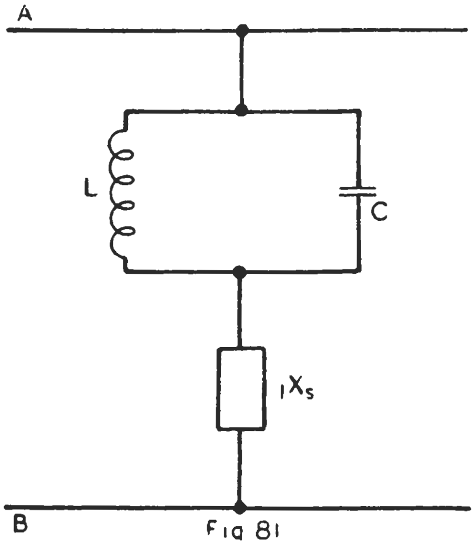


Fig 81

In certain special cases it may be necessary to employ a combination of reactances such that the combination functions as a rejector at one particular frequency and an acceptor at another particular frequency.

Suppose, for example, that it is desired to arrange for a virtual short-circuit across the line A and B, Fig. 81, at a particular unwanted frequency but to have the maximum possible impedance in shunt at the frequency that is being transmitted along A and B. The inductance L and capacity C must be so proportioned that they form a rejector at the frequency of transmission. The series reactance X_s must then be made equal and opposite to the series reactance presented by the rejector at the unwanted frequency.

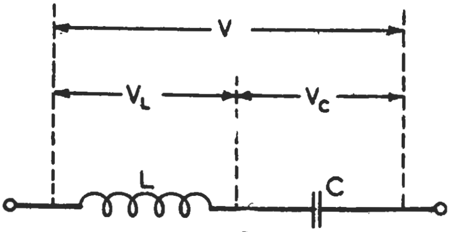


Fig 82

For a constant voltage at the wanted frequency across an acceptor combination the voltages at the wanted frequency across the inductance and capacity respectively are independent of the L/C ratio and are a function of the wanted and unwanted frequencies. It should be understood that by the term "unwanted frequency" is implied the resonant frequency of the acceptor.

With reference to Fig. 82,

$$V_l = V\left[\frac{1}{1-\left(\frac{F_o}{F}\right)^2}\right] \qquad \text{(59)}$$

$$V_c = V\left[\frac{1}{1-\left(\frac{F}{F_o}\right)^2}\right] \qquad \text{(60)}$$

where F=wanted frequency.
F_o=unwanted frequency (i.e. resonant frequency of acceptor).

The derivation of the above formulae is given in Appendix A, page 51.

APPENDIX A

DERIVATIONS OF FORMULAE

Section D, page 9. Series to Parallel Equivalent Impedance Conversion.

Let the parallel equivalent of R_s+jX_s be $R_p//jX_p$. The impedance combinations will be equivalent only if the admittances are equal. Thus,

$$\frac{1}{R_p}+\frac{1}{jX_p}=\frac{1}{R_s+jX_s} \quad \text{(1A)}$$

$$\therefore \quad \frac{1}{R_p}-\frac{j}{X_p}=\frac{R_s-jX_s}{(R_s+jX_s)(R_s-jX_s)} \quad \text{(2A)}$$

$$=\frac{R_s-jX_s}{R_s^2+X_s^2}$$

$$=\frac{R_s}{R_s^2+X_s^2}-\frac{jX_s}{R_s^2+X_s^2} \quad \text{(3A)}$$

From equation (3A) it follows that,

$$\frac{1}{R_p}=\frac{R_s}{R_s^2+X_s^2} \quad \text{(4A)}$$

and

$$\frac{1}{X_p}=\frac{X_s}{R_s^2+X_s^2} \quad \text{(5A)}$$

whence

$$R_p=\frac{R_s^2+X_s^2}{R_s}=R_s\left(1+\frac{X_s^2}{R_s^2}\right) \quad \text{(6A)}$$

and

$$X_p=\frac{R_s^2+X_s^2}{X_s}=X_s\left(1+\frac{R_s^2}{X_s^2}\right) \quad \text{(7A)}$$

Section D, page 10. Alternative Values of X_s for Given Value of X_p, R_s being Constant.

From equation (7A),

$$\frac{R_s^2+X_s^2}{X_s}=X_p$$

$$\therefore \quad X_s^2+R_s^2=X_pX_s \quad \text{(8A)}$$

$$\therefore \quad X_s^2-X_pX_s=-R_s^2 \quad \text{(9A)}$$

$$\therefore \quad X_s^2-X_pX_s+\left(\frac{X_p}{2}\right)^2=\frac{X_p^2}{4}-R_s^2 \quad \text{(10A)}$$

$$\therefore \quad \left(X_s-\frac{X_p}{2}\right)^2=\frac{X_p^2}{4}-R_s^2 \quad \text{(11A)}$$

$$\therefore \quad X_s-\frac{X_p}{2}=\pm\sqrt{\frac{X_p^2}{4}-R_s^2} \quad \text{(12A)}$$

$$\therefore \quad X_s=\frac{X_p}{2}\pm\sqrt{\left(\frac{X_p}{2}+R_s\right)\left(\frac{X_p}{2}-R_s\right)} \quad \text{(13A)}$$

By inspection of equation (13A) it will be seen that if $X_p=2R_s$, then X_s is single-valued and is equal to R_s. This proves the case of exception mentioned on page 10.

Section D, page 10. Parallel to Series Equivalent Impedance Conversion.

From equation (1A),

$$\frac{1}{R_s+jX_s}=\frac{1}{R_p}+\frac{1}{jX_p}$$

$$=\frac{R_p+jX_p}{jR_pX_p} \quad \text{(14A)}$$

$$\therefore \quad R_s+jX_s=\frac{jR_pX_p}{R_p+jX_p}$$

$$=\frac{jR_pX_p(R_p-jX_p)}{(R_p+jX_p)(R_p-jX_p)}$$

$$=\frac{R_pX_p^2+jR_p^2X_p}{R_p^2+X_p^2} \quad \text{(15A)}$$

From equation (15A), it follows that,

$$R_s=\frac{R_pX_p^2}{R_p^2+X_p^2}=\frac{R_p}{1+R_p^2/X_p^2} \quad \text{(16A)}$$

and

$$X_s=\frac{R_p^2X_p}{R_p^2+X_p^2}=\frac{X_p}{1+X_p^2/R_p^2} \quad \text{(17A)}$$

Section D, page 11. Alternative Values of X_p for Given Value of X_s, R_p being Constant.

From equation (17A),

$$\frac{R_p^2X_p}{R_p^2+X_p^2}=X_s$$

$$\therefore \quad R_p^2X_s+X_p^2X_s=R_p^2X_p \quad \text{(18A)}$$

$$\therefore \quad X_p^2X_s-R_p^2X_p=-R_p^2X_s \quad \text{(19A)}$$

$$\therefore \quad X_p^2-\frac{R_p^2}{X_s}X_p=-R_p^2 \quad \text{(20A)}$$

$$\therefore \quad X_p^2-\frac{R_p^2}{X_s}X_p+\left(\frac{R_p^2}{2X_s}\right)^2=\frac{R_p^4}{4X_s^2}-R_p^2 \quad \text{(21A)}$$

$$\therefore \quad \left(X_p-\frac{R_p^2}{2X_s}\right)^2=\frac{R_p^4}{4X_s^2}-R_p^2 \quad \text{(22A)}$$

$$\therefore \quad X_p-\frac{R_p^2}{2X_s}=\pm\sqrt{\frac{R_p^4}{4X_s^2}-R_p^2}$$

$$=\pm\sqrt{\frac{R_p^4-4X_s^2R_p^2}{4X_s^2}}$$

$$=\pm\frac{R_p}{2X_s}\sqrt{R_p^2-4X_s^2} \quad \text{(23A)}$$

$$\therefore \quad X_p=\frac{R_p^2}{2X_s}\pm\frac{R_p}{2X_s}\sqrt{R_p^2-4X_s^2}$$

$$=\frac{R_p}{2X_s}\left[R_p\pm\sqrt{(R_p+2X_s)(R_p-2X_s)}\right] \quad \text{(24A)}$$

If $X_s = \frac{1}{2}R_p$ is substituted in equation (24A), then $X_p = R_p$, which proves the case of exception mentioned on page 11.

Section E. page 14, Design Formulae. Downwards L Transducer.

Referring to Fig. 16, X_2 is equal and opposite in value to the equivalent series reactance of R_A and X_1 in parallel.

$$\therefore X_2 = -\frac{R_A^2 X_1}{R_A^2 + X_1^2} \text{ from equation (11), page 10} \quad \text{(25A)}$$

R_B is the equivalent series resistance of R_A and X_1 in parallel.

From equation (7), page 10,

$$R_A R_B = -X_1 X_2$$

$$\therefore X_1 = -\frac{R_A R_B}{X_2} \quad \text{(26A)}$$

Substitute $X_1 = -\frac{R_A R_B}{X_2}$ in equation (25A),

$$\therefore X_2 = \frac{R_A^3 R_B / X_2}{R_A^2 + \frac{R_A^2 R_B^2}{X_2^2}}$$

$$= \frac{R_A^3 R_B}{X_2 (R_A^2 X_2^2 + R_A^2 R_B^2) / X_2^2}$$

$$= \frac{R_A^3 R_B X_2}{R_A^2 X_2^2 + R_A^2 R_B^2} \quad \text{(27A)}$$

From equation (27A),

$$R_A^2 X_2^2 + R_A^2 R_B^2 = R_A^3 R_B \quad \text{(28A)}$$

$$\therefore X_2^2 + R_B^2 = R_A R_B \quad \text{(29A)}$$

$$\therefore X_2 = \pm\sqrt{R_A R_B - R_B^2} \quad \text{(30A)}$$

Let $R_A/R_B = m$

$$\therefore R_A = mR_B$$

Substitute $R_A = mR_B$ in equation (30A)

$$\therefore X_2 = \pm\sqrt{mR_B^2 - R_B^2}$$

$$= \pm R_B\sqrt{m-1} \quad \text{(31A)}$$

Substitute $X_2 = \pm\sqrt{R_A R_B - R_B^2}$ in equation (26A),

$$\therefore X_1 = \mp \frac{R_A R_B}{\sqrt{R_A R_B - R_B^2}} \quad \text{(32A)}$$

Substitute $R_A = mR_B$ in equation (32A),

$$\therefore X_1 = \pm \frac{mR_B^2}{R_B\sqrt{m-1}}$$

$$= \mp \frac{mR_B}{\sqrt{m-1}} \quad \text{(33A)}$$

Section E, page 17, Design Formulae. Upwards L Transducer.

Referring to Fig. 24, the equivalent parallel resistance of $R_B + jX_2$ is equal to R_A.

$$\therefore \frac{R_B^2 + X_2^2}{R_B} = R_A \text{ (from equation 3, page 9)} \quad \text{(34A)}$$

X_1 is equal and opposite in value to the equivalent parallel reactance of $R_B + jX_2$.

$$\therefore \frac{R_B^2 + X_2^2}{X_2} = -X_1 \text{ (from equation 4, page 9)} \quad \text{(35A)}$$

$$\therefore R_B^2 + X_2^2 = -X_1 X_2 \quad \text{(36A)}$$

Substitute $R_B^2 + X_2^2 = -X_1 X_2$ in (34A),

$$\therefore \frac{-X_1 X_2}{R_B} = R_A \quad \text{(37A)}$$

$$\therefore -X_1 X_2 = R_A R_B \quad \text{(38A)}$$

From equation (34A),

$$R_B^2 + X_2^2 = R_A R_B \quad \text{(39A)}$$

$$\therefore X_2 = \pm\sqrt{R_A R_B - R_B^2} \quad \text{(40A)}$$

Let $\frac{R_A}{R_B} = m$,

$$\therefore R_A = mR_B.$$

Substitute $R_A = mR_B$ in equation (40A),

$$\therefore X_2 = \pm\sqrt{mR_B^2 - R_B^2}$$

$$= \pm R_B\sqrt{m-1} \quad \text{(41A)}$$

Substitute $X_2 = \pm R_B\sqrt{m-1}$ in (35A),

$$\therefore X_1 = \mp \frac{R_B^2 + R_B^2(m-1)}{R_B\sqrt{m-1}}$$

$$= \mp \frac{mR_B^2}{R_B\sqrt{m-1}}$$

$$= \mp \frac{mR_B}{\sqrt{m-1}} \quad \text{(42A)}$$

Section F, page 20. T-type Transforming Networks.

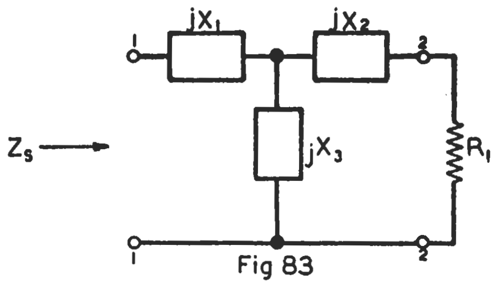


Fig 83

Equations (27) and (28), page 20, can be derived as follows :—

The impedance looking into the terminals 1, 1 (Fig. 83) when R_1 is connected to the terminals 2, 2 is,

$$Z_s = jX_1 + \frac{jX_3(R_1 + jX_2)}{R_1 + jX_2 + jX_3}$$

$$= jX_1 + \frac{jX_3(R_1 + jX_2)}{R_1 + j(X_2 + X_3)}$$

$$= \frac{jR_1X_1 - X_1(X_2+X_3) + jX_3(R_1 + jX_2)}{R_1 + j(X_2 + X_3)} \quad \text{(43A)}$$

Let $Z_s = R_2$,

$$\therefore R_2 = \frac{jR_1X_1 - X_1(X_2+X_3) + jX_3(R_1+jX_2)}{R_1 + j(X_2+X_3)} \quad (44A)$$

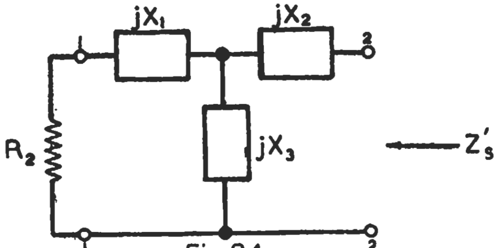


Fig 84

The impedance looking into the terminals 2, 2 (Fig. 84), when R_2 is connected to the terminals 1, 1 is,

$$Z'_s = jX_2 + \frac{jX_3(R_2+jX_1)}{R_2+jX_1+jX_3}$$

$$= jX_2 + \frac{jX_3(R_2+jX_1)}{R_2+j(X_1+X_3)}$$

$$= \frac{jR_2X_2 - X_2(X_1+X_3) + jX_3(R_2+jX_1)}{R_2+j(X_1+X_3)} \quad (45A)$$

Let $Z'_s = R_1$,

$$\therefore R_1 = \frac{jR_2X_2 - X_2(X_1+X_3) + jX_3(R_2+jX_1)}{R_2+j(X_1+X_3)} \quad (46A)$$

From equation (44A),

$$R_1R_2 + jR_2(X_2+X_3) = jR_1X_1 - X_1(X_2+X_3) + jX_3(R_1+jX_2) \quad \text{.......(47A)}$$

From equation (46A),

$$R_1R_2 + jR_1(X_1+X_3) = jR_2X_2 - X_2(X_1+X_3) + jX_3(R_2+jX_1) \quad \text{.......(48A)}$$

Subtracting equation (48A) from equation (47A),

$$jR_2(X_2+X_3) - jR_1(X_1+X_3) = jR_1(X_1+X_3) - jR_2(X_2+X_3) \quad \text{.......(49A)}$$

$$\therefore 2R_2(X_2+X_3) = 2R_1(X_1+X_3) \quad \text{.......(50A)}$$

$$\therefore \frac{R_2}{R_1} = \frac{X_1+X_3}{X_2+X_3} \quad \text{.......(51A)}$$

Adding equations (47A) and (48A),

$$2R_1R_2 = -2X_1X_3 - 2X_2X_3 - 2X_1X_2 \quad \text{......(52A)}$$

$$\therefore -R_1R_2 = X_1X_3 + X_2X_3 + X_1X_2$$
$$= X_3^2 + X_1X_3 + X_2X_3 + X_1X_2 - X_3^2$$
$$= (X_1+X_3)(X_2+X_3) - X_3^2 \quad \text{......(53A)}$$

$$\therefore R_1R_2 = X_3^2 - (X_1+X_3)(X_2+X_3) \quad \text{......(54A)}$$

From equation (51A),

$$X_1 + X_3 = \frac{R_2(X_2+X_3)}{R_1} \quad \text{.......(55A)}$$

Substitute $X_1 + X_3 = \dfrac{R_2(X_2+X_3)}{R_1}$ in equation (54A),

$$\therefore R_1R_2 = X_3^2 - \frac{R_2(X_2+X_3)^2}{R_1} \quad \text{.........(56A)}$$

$$\therefore (X_2+X_3)^2 = \frac{R_1X_3^2}{R_2} - R_1^2 \quad \text{.........(57A)}$$

$$\therefore X_2 + X_3 = \pm X_3\sqrt{\frac{R_1}{R_2} - \frac{R_1^2}{X_3^2}} \quad \text{.......(58A)}$$

$$\therefore X_2 = -X_3\left(1 \pm \sqrt{\frac{R_1}{R_2} - \frac{R_1^2}{X_3^2}}\right) \quad \text{.......(59A)}$$

By substituting $X_2 + X_3 = \dfrac{R_1(X_1+X_3)}{R_2}$, from equation (51A) in equation (54A), it can be shown that,

$$X_1 = -X_3\left(1 \pm \sqrt{\frac{R_2}{R_1} - \frac{R_2^2}{X_3^2}}\right) \quad \text{..........(60A)}$$

Section G, page 23. π-type Transforming Networks.

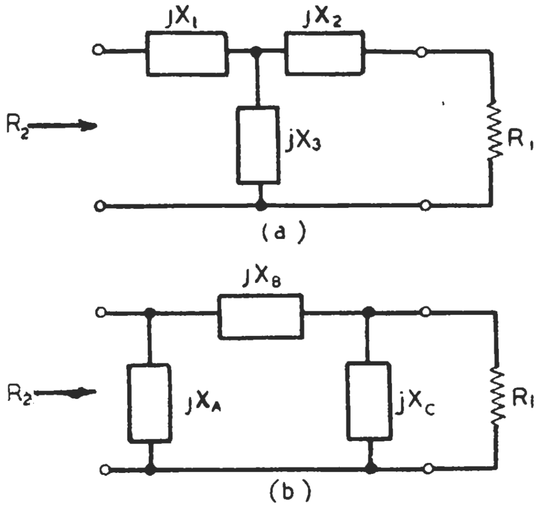


Fig 85

If the T and π transforming networks of Figs. 85 (a) and 85 (b) are equivalent, it can be shown that,

$$X_1 = \frac{X_A X_B}{X_A + X_B + X_C}$$

$$X_2 = \frac{X_B X_C}{X_A + X_B + X_C}$$

$$X_3 = \frac{X_A X_C}{X_A + X_B + X_C}$$

Proof of the above relations will be found in standard text-books.

Equations (29) and (30), page 23, can be derived as follows :—

Substitute in equation (51A) the equivalent expressions given above,

$$\therefore \frac{R_2}{R_1} = \frac{X_A X_B + X_A X_C}{X_B X_C + X_A X_C} \quad \text{.................. (61A)}$$

Substitute also in equation (54A),

$$\therefore R_1 R_2 = \frac{X_A{}^2 X_C{}^2 - (X_A X_B + X_A X_C)(X_B X_C + X_A X_C)}{(X_A + X_B + X_C)^2}$$

$$= \frac{-X_A X_B X_C}{X_A + X_B + X_C} \quad \text{...............(62A)}$$

From equation (61A),

$$R_2 X_B X_C + R_2 X_A X_C = R_1 X_A X_B + R_1 X_A X_C \text{...(63A)}$$

$$\therefore X_A (R_1 X_C - R_2 X_C + R_1 X_B) = R_2 X_B X_C \text{...(64A)}$$

$$\therefore X_A = \frac{R_2 X_B X_C}{R_1 X_C - R_2 X_C + R_1 X_B} \quad \text{............ (65A)}$$

From equation (62A),

$$R_1 R_2 X_A + R_1 R_2 X_B + R_1 R_2 X_C = -X_A X_B X_C \quad \text{(66A)}$$

$$\therefore X_A (R_1 R_2 + X_B X_C) = -R_1 R_2 X_B - R_1 R_2 X_C \quad \text{(67A)}$$

$$\therefore X_A = \frac{-R_1 R_2 X_B - R_1 R_2 X_C}{R_1 R_2 + X_B X_C} \quad \text{...............(68A)}$$

From equations (65A) and (68A),

$$\frac{R_2 X_B X_C}{R_1 X_C - R_2 X_C + R_1 X_B} = \frac{-R_1 R_2 X_B - R_1 R_2 X_C}{R_1 R_2 + X_B X_C} \text{(69A)}$$

$$\therefore \frac{X_B X_C}{R_1 X_C - R_2 X_C + R_1 X_B} = \frac{-R_1 X_B - R_1 X_C}{R_1 R_2 + X_B X_C} \text{...(70A)}$$

$$\therefore R_1 R_2 X_B X_C + X_B{}^2 X_C{}^2 = - R_1{}^2 X_B X_C + R_1 R_2 X_B X_C - R_1{}^2 X_B{}^2 - R_1{}^2 X_C{}^2 + R_1 R_2 X_C{}^2 - R_1{}^2 X_B X_C \text{.....................(71A)}$$

$$\therefore X_B{}^2 X_C{}^2 = - R_1{}^2 X_B{}^2 - 2R_1{}^2 X_B X_C - R_1{}^2 X_C{}^2 + R_1 R_2 X_C{}^2 \text{......................(72A)}$$

$$\therefore X_B{}^2 X_C{}^2 = -R_1{}^2 (X_B + X_C)^2 + R_1 R_2 X_C{}^2 \text{...(73A)}$$

$$\therefore R_1{}^2 (X_B + X_C)^2 = X_C{}^2 (R_1 R_2 - X_B{}^2) \text{.....(74A)}$$

$$\therefore R_1 X_C + R_1 X_B = \pm X_C \sqrt{R_1 R_2 - X_B{}^2} \text{...(75A)}$$

$$\therefore X_C (R_1 \pm \sqrt{R_1 R_2 - X_B{}^2}) = - R_1 X_B \text{...(76A)}$$

$$\therefore X_C = \frac{-R_1 X_B}{R_1 \pm \sqrt{R_1 R_2 - X_B{}^2}} \quad \text{.............. (77A)}$$

By solving equations (63A) and (66A) for X_C and proceeding on similar lines to the above, it can be shown that,

$$X_A = \frac{-R_2 X_B}{R_2 \pm \sqrt{R_1 R_2 - X_B{}^2}} \quad \text{.................. (78A)}$$

Section K, page 43. Pass Reactance of Rejector.

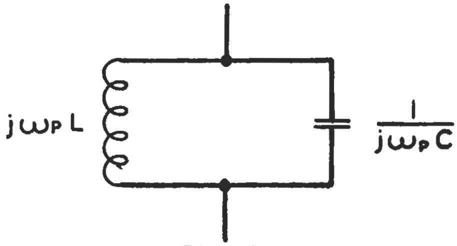


Fig 86

The pass reactance (i.e. the equivalent series reactance at the pass frequency) of a rejector is,

$$X \text{ pass} = \frac{X_L X_C}{X_L + X_C} \quad \text{.................(79A)}$$

where $X_L = \omega_p L$

$X_C = - 1/\omega_p C$

$\omega_p = 2\pi \times$ pass frequency.

$$\therefore \quad X \text{ pass} = - \frac{\omega_p L \times 1/\omega_p C}{\omega_p L - 1/\omega_p C}$$

$$= - \frac{L/C}{\dfrac{\omega_p{}^2 LC - 1}{\omega_p C}}$$

$$= - \frac{\omega_p L}{\omega_p{}^2 LC - 1} \quad \text{...........(80A)}$$

Now, $\omega_r L = 1/\omega_r C$......................(81A)

where $\omega_r = 2\pi \times$ reject frequency.

$$\therefore \quad L = \frac{1}{\omega_r{}^2 C} \quad \text{.......................(82A)}$$

and

$$LC = \frac{1}{\omega_r{}^2} \quad \text{....................... (83A)}$$

Substitute $L = \dfrac{1}{\omega_r{}^2 C}$ and $LC = \dfrac{1}{\omega_r{}^2}$ in equation (80A)

$$\therefore \quad X \text{ pass} = - \frac{\omega_p / \omega_r{}^2 C}{\omega_p{}^2/\omega_r{}^2 - 1} \quad \text{..........(84A)}$$

$$= - \frac{\omega_p}{(\omega_p{}^2 - \omega_r{}^2)\, C} \quad \text{.........(85A)}$$

But

$\omega_p = 2\pi F_p$

$\omega_r = 2\pi F_r$

where F_p=pass frequency

F_r=reject frequency

Substitute $\omega_p = 2\pi F_p$ and $\omega_r = 2\pi F_r$ in equation (85A),

$$\therefore \quad X \text{ pass} = - \frac{F_p}{2\pi\,(F_p{}^2 - F_r{}^2)\, C} \quad \text{.....(86A)}$$

Expressing F_p and F_r in kc/s and C in $\mu\mu F$,

$$X \text{ pass} = - \frac{10^9 F_p}{2\pi C\,(F_p + F_r)(F_p - F_r)} \quad \text{..........(87A)}$$

Section K, page 44. Components of Pass Current in the Condenser and Inductance Branches of a Rejector.

Condenser Branch :—

$$I_c = I_p\left(\frac{\omega_p L}{\omega_p L - 1/\omega_p C}\right) \text{ ....(88A)}$$

where I_p = total pass current.

I_c = current in condenser branch.

$\omega_p L$ = reactance of inductance at pass frequency.

$-1/\omega_p C$ = reactance of condenser at pass frequency.

$\omega_p = 2\pi \times$ pass frequency.

From equation (88A),

$$I_c = I_p\left(\frac{\omega_p{}^2 LC}{\omega_p{}^2 LC - 1}\right) \text{ ....(89A)}$$

But $LC = \dfrac{1}{\omega_r{}^2}$

where $\omega_r = 2\pi \times$ reject frequency.

$$\therefore I_c = I_p\left(\frac{\omega_p{}^2/\omega_r{}^2}{\omega_p{}^2/\omega_r{}^2 - 1}\right)$$

$$= I_p\left[\frac{1}{1-\left(\dfrac{\omega_r}{\omega_p}\right)^2}\right]$$

$$= I_p\left[\frac{1}{1-\left(\dfrac{F_r}{F_p}\right)^2}\right] \text{ ....(90A)}$$

where F_r = reject frequency

F_p = pass frequency.

Inductance Branch :—

$$I_l = I_p\left(\frac{-1/\omega_p C}{\omega_p L - 1/\omega_p C}\right)$$

$$= I_p\left(\frac{1}{1-\omega_p{}^2 LC}\right)$$

$$= I_p\left[\frac{1}{1-\left(\dfrac{\omega_p}{\omega_r}\right)^2}\right]$$

$$= I_p\left[\frac{1}{1-\left(\dfrac{F_p}{F_r}\right)^2}\right] \text{ ....(91A)}$$

Section L, page 45. Acceptors. Values of L and C for Zero Reactance at a Frequency F_o and a Reactance of X'_c at a Frequency F.

Since the LC combination resonates at F_o,

$$\omega_o L = \frac{1}{\omega_o C} \text{ ....(92A)}$$

where $\omega_o = 2\pi F_o$.

At the frequency F the reactance of the LC combination $= \omega L - \dfrac{1}{\omega C}$

where $\omega = 2\pi F$.

$$\text{Let } X'_c = \omega L - \frac{1}{\omega C} \text{ ....(93A)}$$

where X'_c is the reactance of the equivalent condenser at some frequency $\omega/2\pi$.

From equation (93A),

$$X'_c = \frac{\omega^2 LC - 1}{\omega C} \text{ .... (94A)}$$

From equation (92A), $LC = \dfrac{1}{\omega_o{}^2}$

Substitute $LC = \dfrac{1}{\omega_o{}^2}$ in equation (94A),

$$\therefore X_c' = \frac{\omega^2/\omega_c{}^2 - 1}{\omega C}$$

$$= \frac{\omega^2 - \omega_o{}^2}{\omega_o{}^2\,\omega C} \text{ ....(95A)}$$

whence

$$C = \frac{\omega^2 - \omega_o{}^2}{\omega_o{}^2\,\omega X'_c} \text{ ....(96A)}$$

It will be observed in equation (95A) that if $\omega < \omega_o$, then X'_c is negative; that is, the acceptor combination is the equivalent of a condenser.

Section L, page 46. Acceptors. Voltage Components at Wanted Frequency.

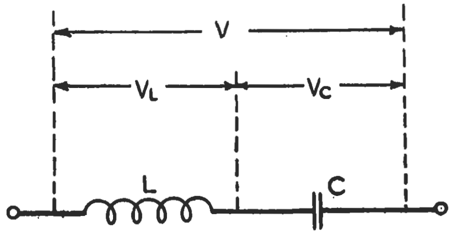


Fig 87

$$V_l = V\left(\frac{\omega L}{\omega L - 1/\omega C}\right)$$

$$= V\left(\frac{\omega^2 LC}{\omega^2 LC - 1}\right) \text{ ....(97A)}$$

where $\omega = 2\pi \times$ wanted frequency.

Now, $LC = \dfrac{1}{\omega_o{}^2}$

where $\omega_o = 2\pi \times$ unwanted frequency (i.e. resonant frequency of acceptor).

Substitute $LC = \dfrac{1}{\omega_0^2}$ in equation (97A).

$$\therefore V_l = V \frac{\omega^2/\omega_0^2}{\omega^2/\omega_0^2 - 1}$$

$$= V\left[\frac{1}{1-\left(\dfrac{\omega_0}{\omega}\right)^2}\right]$$

$$= V\left[\frac{1}{1-\left(\dfrac{F_0}{F}\right)^2}\right] \quad \text{..............(98A)}$$

where F_0 = unwanted frequency

F = wanted frequency.

Similarly,

$$V_c = V\left(\frac{-1/\omega C}{\omega L - 1/\omega C}\right)$$

$$\neq V\left(\frac{1}{1-\omega^2 LC}\right) \quad \text{.....................(99A)}$$

Substitute $LC = \dfrac{1}{\omega_0^2}$ in equation (99A).

$$\therefore V_c = V\left[\frac{1}{1-\left(\dfrac{\omega}{\omega_0}\right)^2}\right]$$

$$= V\left[\frac{1}{1-\left(\dfrac{F}{F_0}\right)^2}\right] \quad \text{............(100A)}$$

APPENDIX B

PROBLEMS

(1) Calculate the equivalent parallel values of: (*a*) 6.6 − j40, (*b*) 110 + j254, (*c*) 78 + j15 ohms.

(2) Calculate the equivalent series values of: (*a*) 1370//−j306, (*b*) 172//j1408, (*c*) 200//−j105 ohms.

(3) What are the inductance and capacity values, in μH and μμF respectively, required for the following simple L-type transducers employing shunt capacitive arms?

(*a*) Transforming 150 ohms resistance to 80 ohms resistance. Frequency 668 kc/s.

(*b*) Transforming 45 ohms resistance to 80 ohms resistance. Frequency 1474 kc/s.

(*c*) Transforming 134 ohms resistance to 300 ohms resistance. Frequency 970 kc/s.

(4) What reactance will, if connected in series with a resistance of 68 ohms, provide an equivalent parallel resistance of 750 ohms?

(5) What are the alternative values of reactance which, if connected in series with a resistance of 115 ohms, will provide an equivalent parallel reactance of +280 ohms?

(6)

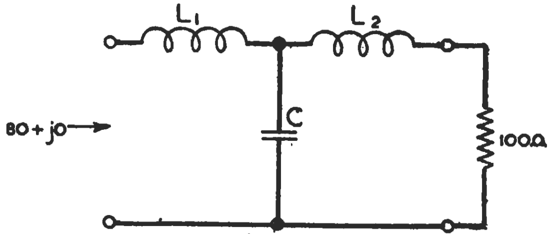


Fig 88

Calculate the values of L_1, L_2 and C for the T-type transducer illustrated in Fig. 88, given that the frequency is 1200 kc/s and the mid-shunt resistance is 500 ohms.

(7)

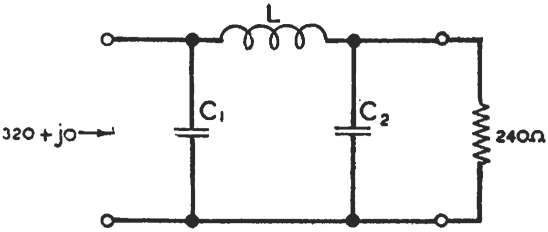


Fig 89

Given that the π-type transducer illustrated in Fig. 89 operates at a frequency of 1013 kc/s and that C_2=785 μμF, what is the capacity of C_1?

(8) Relating to Problem (7), if the power input to the network is 50 kW, what are the values of (*a*) voltage across C_1, (*b*) voltage across C_2, (*c*) current in L? Neglect component losses.

(9) If the driving-point impedance of an aerial is 50 − j200 ohms, what are the values of (*a*) carrier current at the driving point, (*b*) carrier volts to ground at the driving point, given that the carrier power is 20 kW?

(10) Sketch A.T.H. matching circuits and calculate the component reactances for the cases detailed below. In each case restrict the circuit to the simplest possible form consistent with specified conditions:—

(*a*) Aerial impedance Z_s=762 − j420 ohms. Unbalanced feeder of 300 ohms characteristic impedance. Base the circuit design upon the series aspect of the aerial impedance.

(*b*) Aerial impedance 30+j10 ohms. Unbalanced feeder of 300 ohms characteristic impedance. Base the circuit design upon the series aspect of the aerial impedance.

(*c*) As for (*b*), but base the circuit design upon the parallel aspect of the aerial impedance.

(11) Referring to (10) (*c*), what are the peak voltages across the condensers at 100 per cent. modulation, given that the carrier power is 50 kW?

(12)

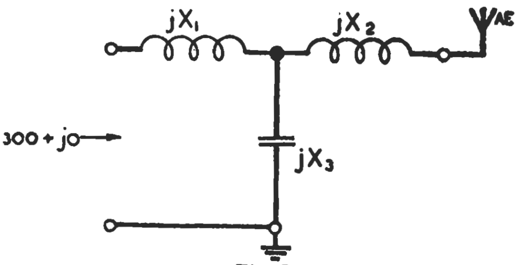


Fig 90

An A.T.H. circuit of the form illustrated in Fig. 90 is to be set up to match an aerial of $Z_s = 1000 - j2000$ ohms to an unbalanced feeder of 300 ohms characteristic impedance. Given that it is a necessary condition that the carrier voltage across the condenser shall be 5000 volts R.M.S. at a carrier power of 10 kW, calculate the reactance values marked in Fig. 90.

(13) An 804 kc/s rejector is to be installed immediately below the driving point of an aerial of a 50 kW, 668 kc/s installation. Given that the rejector condenser is to be of 8000 $\mu\mu$F and that the aerial impedance is $Z_s = 100 + j50$ ohms, calculate : (*a*) value of rejector inductance, (*b*) current at pass frequency in the condenser, (*c*) pass reactance of rejector.

(14) What are the L and C values for an acceptor to the 2nd harmonic of 804 kc/s if it be required that the LC combination shall present a reactance of −150 ohms at the fundamental frequency?

ANSWERS

(1) (*a*) 249.1//−j41.1, (*b*) 696.5//j301.8, (*c*) 80.9//j420.7 ohms.

(2) (*a*) 65.1−j291.4, (*b*) 169.4+j20.7, (*c*) 43.2 −j82.3 ohms.

(3) (*a*) 17.8 μH, 1485 $\mu\mu$F ; (*b*) 4.3 μH, 1189 $\mu\mu$F ; (*c*) 24.4 μH, 608.1 $\mu\mu$F.

(4) ±215.4 ohms.

(5) +219.9 or +60.2 ohms.

(6) $L_1 = 24.3$ μH., $L_2 = 26.5$ μH., C=1137 $\mu\mu$F.

(7) 736.5 $\mu\mu$F.

(8) (*a*) 4000 volts, (*b*) 3460 volts, (*c*) 22.6 amps.

(9) (*a*) 20 amps, (*b*) 4120 volts.

(10) (*a*)

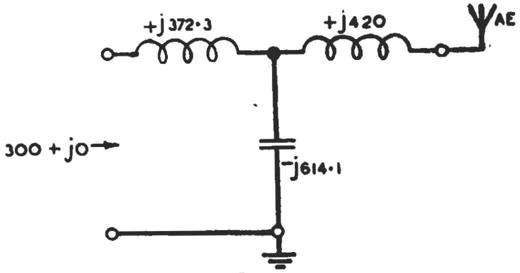


Fig 91

(*b*)

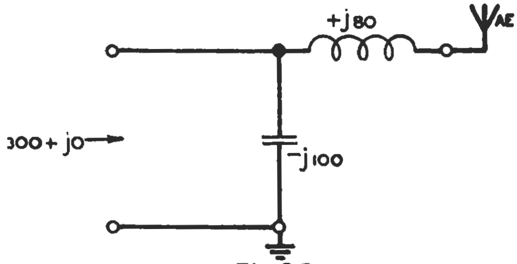


Fig 92

(*c*)

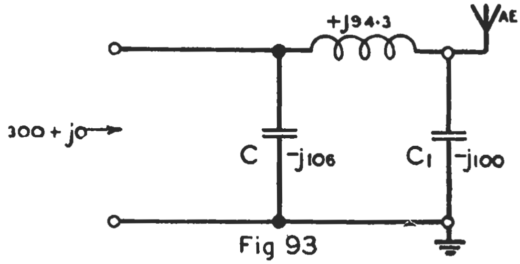


Fig 93

(11) With reference to Fig. 93,
Peak Voltage across C = 10,950.
" " " C_1=3650.

(12) With reference to Fig. 90,

	Alternative answer
$X_1 = +812.4$ ohms	$X_1 = +812.4$ ohms
$X_2 = +775$ "	$X_2 = +3225$ "
$X_3 = -1685$ "	$X_3 = -635.6$ "

(13) (*a*) L=4.9 μH, (*b*) I_c=49.8 amps. R.M.S., (*c*) +66.8 ohms.

(14) C=989.5 $\mu\mu$F., L=9.9 μH.

APPENDIX C

LIST OF FORMULAE

Series and Parallel Impedance Equivalents.

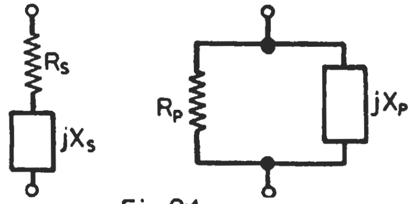


Fig 94

$$R_p = R_s\left(1 + \frac{X_s^2}{R_s^2}\right) \quad \text{(1C)}$$

$$X_p = X_s\left(1 + \frac{R_s^2}{X_s^2}\right) \quad \text{(2C)}$$

$$R_s = \frac{R_p}{1 + R_p^2/X_p^2} \quad \text{(3C)}$$

$$X_s = \frac{X_p}{1 + X_p^2/R_p^2} \quad \text{(4C)}$$

$$R_pR_s = X_pX_s \quad \text{(5C)}$$

Alternative Values of X_s for Given Value of X_p, R_s being Constant.

$$X_s = \frac{X_p}{2} \pm \sqrt{\left(\frac{X_p}{2} + R_s\right)\left(\frac{X_p}{2} - R_s\right)} \quad \text{(6C)}$$

Alternative Values of X_p for Given Value of X_s, R_p being Constant.

$$X_p = \frac{R_p}{2X_s}\left[R_p \pm \sqrt{(R_p + 2X_s)(R_p - 2X_s)}\right] \quad \text{(7C)}$$

L-type Transducers.

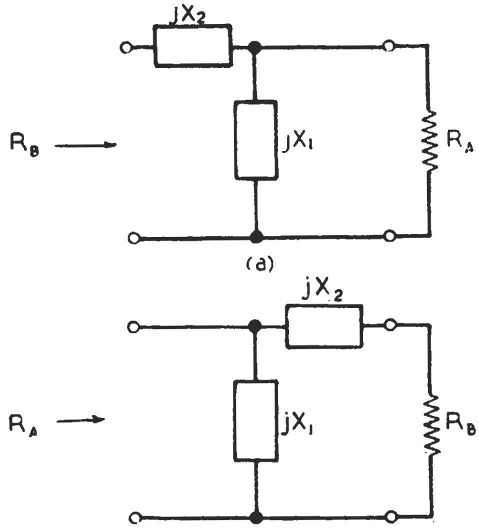


Fig 95

$$X_1 = \pm\frac{mR_B}{\sqrt{m-1}} \quad \text{(8C)}$$

$$X_2 = \mp R_B\sqrt{m-1} \quad \text{(9C)}$$

where $m = R_A/R_B$

Note :—m is the ratio of the *larger* resistance value to the *smaller* resistance value.

T-type Transducers.

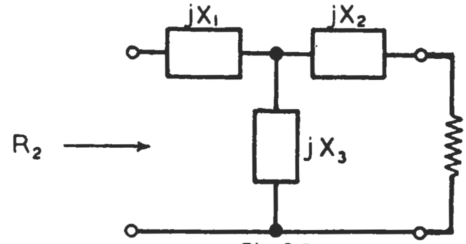


Fig 96

$$X_1 = -X_3\left(1 \pm \sqrt{\frac{R_2}{R_1} - \frac{R_2^2}{X_3^2}}\right) \quad \text{(10C)}$$

$$X_2 = -X_3\left(1 \pm \sqrt{\frac{R_1}{R_2} - \frac{R_1^2}{X_3^2}}\right) \quad \text{(11C)}$$

Note :—In a quarter-wave network $X_1 = X_2 = -X_3 = \pm\sqrt{R_1R_2}$ ..(12C)

π-type Transducers.

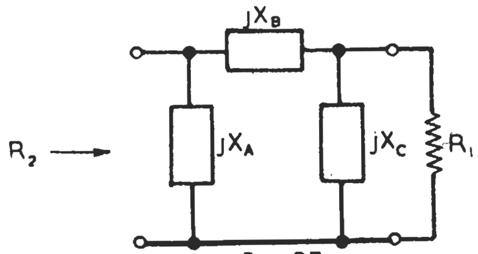


Fig 97

$$X_A = \frac{-R_2X_B}{R_2 \pm \sqrt{R_1R_2 - X_B^2}} \quad \text{(13C)}$$

$$X_C = \frac{-R_1X_B}{R_1 \pm \sqrt{R_1R_2 - X_B^2}} \quad \text{(14C)}$$

Note :—In a quarter-wave network $X_A = X_C = -X_B = \pm\sqrt{R_1R_2}$ .... (15C)

Mutual Coupling. Series Impedance Reflected into Primary Circuit.

$$Z = \frac{(\omega M)^2R_s}{R_s^2 + X_s^2} - j\left[\frac{(\omega M)^2X_s}{R_s^2 + X_s^2}\right] \quad \text{(16C)}$$

where $\omega = 2\pi$ frequency (c/s),

R_s = series resistance of secondary circuit

X_s=series reactance of secondary circuit
M=mutual inductance (H).

If the secondary circuit is adjusted to unity power factor,

$$Z = \frac{(\omega M)^2}{R_s} \quad \text{.....(17C)}$$

Rejectors. Reactance at Pass Frequency.

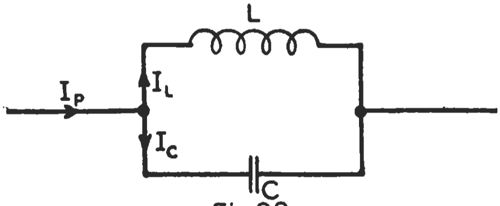


Fig 98

$$X \text{ pass} = \frac{X_L X_C}{X_L + X_C} \quad \text{(due regard being paid to signs)} \text{.....(18C)}$$

where X_L = reactance of L at the pass frequency
X_C = „ „ C „ „ „ „

$$X \text{ pass} = -\frac{10^9 F_p}{2\pi C (F_p + F_r)(F_p - F_r)} \text{ ... (19C)}$$

where C = capacity ($\mu\mu$F)
F_p = pass frequency (kc/s)
F_r = reject frequency (kc/s)

Rejectors. Currents at Pass Frequency in Condenser and Inductance.

$$I_c = I_p \left[\frac{1}{1 - \left(\frac{F_r}{F_p}\right)^2} \right] \text{.....(20C)}$$

$$I_l = I_p \left[\frac{1}{1 - \left(\frac{F_p}{F_r}\right)^2} \right] \text{.....(21C)}$$

$I_p = I_l + I_c$ (due regard being paid to the signs of I_l and I_c) .... (22C)

Rejectors. Total Currents in Condenser and Inductance.

I'_c (total) $= \sqrt{I_c^2 + I_r^2}$ amps. (R.M.S.)...(23C)
I'_l (total) $= \sqrt{I_l^2 + I_r^2}$ amps. (R.M.S.)...(24C)

where I_c=current component at pass frequency in condenser (amps., R.M.S.).
I_l = current component at pass frequency in inductance (amps., R.M.S.).
I_r=circulating current at reject frequency in condenser (amps., R.M.S.).

Acceptors. Values of C and L to accept a Frequency F_o and to present a reactance of X'_c at a Frequency F.

$$C = \frac{\omega^2 - \omega_o^2}{\omega_o^2 \omega X'_c} \quad \text{(F) .....(25C)}$$

$$L = \frac{1}{\omega_o^2 C} \quad \text{(H) .....(26C)}$$

where $\omega = 2\pi F$ (c/s)
$\omega_o = 2\pi F_o$ (c/s)

If $\omega_o = n\omega$ then,

$$C = \frac{1 - n^2}{n^2 \omega X'_c} \quad \text{(F) .....(27C)}$$

Acceptors. Voltage Components at the Wanted Frequency.

Across the inductance,

$$V_l = V \left[\frac{1}{1 - \left(\frac{F_o}{F}\right)^2} \right] \text{.....(28C)}$$

Across the condenser,

$$V_c = V \left[\frac{1}{1 - \left(\frac{F}{F_o}\right)^2} \right] \text{.....(29C)}$$

where V=total voltage at wanted frequency across acceptor.
F_o=unwanted frequency (i.e. resonant frequency of acceptor)
F=wanted frequency

Characteristic Impedance of R.F. Feeders.

Two-wire Balanced Feeder :—

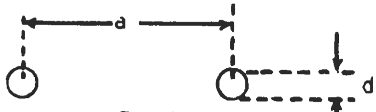


Fig 99

$$Z_o = 276 \log_{10} \frac{2a}{d} \quad \text{.....(30C)}$$

Four-wire Balanced Feeder :—

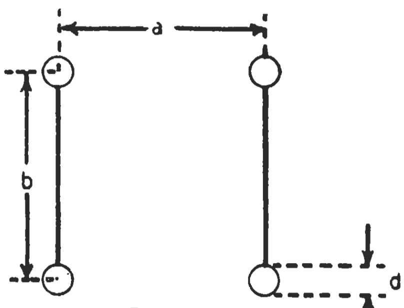


Fig 100

$$Z_o = 138 \log_{10} \left[\frac{2a}{d} \sqrt{\frac{1 + M^2}{M^2}} \right] \text{.....(31C)}$$

where $M = \frac{b}{a}$

Concentric Unbalanced Feeder :—

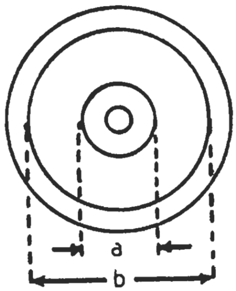


Fig 101

$$Z_0 = 138 \log_{10} \frac{b}{a} \quad \text{(32C)}$$

Five-wire Unbalanced Feeder :—

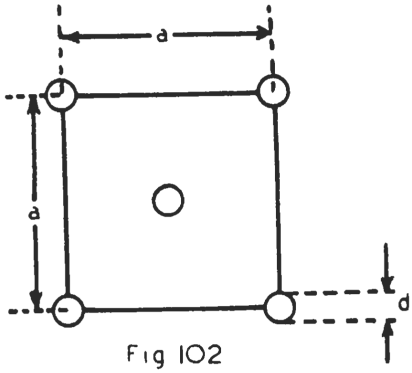


Fig 102

$$Z_0 = 69 \log_{10} \frac{1.414a^3}{d^2\sqrt{da}} \quad \text{(33C)}$$