Attenuators, Pads and Branching Networks

1. Attenuators and Pads are resistive networks designed to introduce known attenuation or insertion loss when operated between specified resistive terminations (usually 600, 140 or 75 ohms). The term "attenuator" is usually applied to a variable network, while the term "pad" is applied to a fixed network. Attenuators and pads can be constructed in lattice, T, π, L or bridged-T forms, the latter four having unbalanced or balanced equivalents. The design formulae which follow are based on either image attenuation or insertion loss according to convenience of use in practice; for a symmetrical network presenting image conditions the two quantities are identical since under such conditions there is no reflection loss.

In the formulae which follow, the symbols Ro1, Ro2, n, a, N, b, are used for parameters defined as follows;

Ro1, Ro2, input, output or terminating resistance.

n = image attenuation in db

$$\left(\text{i.e., } 10 \log_{10} \left\{\frac{\text{Volt-amp input}}{\text{Volt-amp output}}\right\} \text{ when image-terminated}\right)$$

and $a = \text{antilog}_{10} \frac{n}{20}$

N = insertion loss in db with specified terminations (i.e., if the sending termination has emf E and resistance Ro1, and receiving termination is Ro2 with output voltage V2, then insertion loss is

$$20 \log_{10} \left\{\frac{E.}{V2} \; \frac{Ro2}{Ro1 + Ro2}\right\} \text{db.})$$

and $b = \text{antilog} \frac{N}{20}$

The resistor values given in Tables 1 to 12 have been calculated and differenced and are believed to be accurate to four significant figures.

2. The Symmetrical Lattice Attenuator

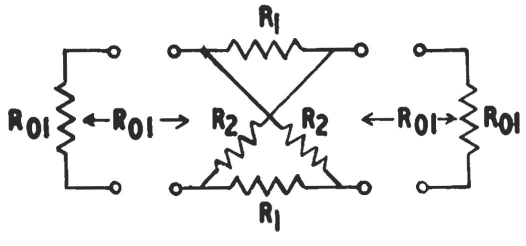


TABLE 1

Values of $R_1 = R_2$ for 600 - ohm symmetrical T attenuator

0.1 to 0.9 db

db	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
	3.454	6.906	10.36	13.81	17.26	20.71	24.16	27.61	31.06

1 to 40 db

db	0	1	2	3	4	5	6	7	8	9
0	0	34.50	68.78	102.6	135.8	168.1	199.4	229.5	258.3	285.7
10	311.7	336.3	359.1	380.5	400.4	418.8	435.8	451.5	465.8	478.9
20	490.9	501.7	511.7	520.7	528.8	536.1	542.7	548.7	554.1	558.9
30	563.2	567.1	570.6	573.7	576.5	579.0	581.3	583.3	585.1	586.7
40	588.1	-	-	-	-	-	-	-	-	-

TABLE 2

Values of R_3 for 600 - ohm symmetrical T attenuator

0.1 to 0.9 db

db	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
	52116	26059	17369	12994	10417	8679	7437	6505	5780

1 to 40 db

db	0	1	2	3	4	5	6	7	8	9
0	∝	5200	2583	1703	1258	986.9	803.2	669.6	567.7	487.1
10	421.6	367.4	321.7	282.8	249.4	220.4	194.9	173.0	153.5	136.4
20	121.2	107.8	95.92	85.38	76.02	67.69	60.30	53.71	47.85	42.63
30	37.99	33.85	30.16	26.88	23.95	21.35	19.02	16.95	15.11	13.47
40	12.00	-	-	-	-	-	-	-	-	-

The corresponding values of R_1 and R_3 for 75 ohm attenuators are given in Tables 3 and 4. For other values of Ro multiply values of Tables 1 and 2 by $\frac{Ro}{600}$.

TABLE 3

Values of $R_1 = R_2$ for 75 - ohm symmetrical T attenuator

0.1 to 0.9 db

db	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
	0.432	0.863	1.045	1.726	2.158	2.590	3.02	3.451	3.882

1 to 20 db

db	0	1	2	3	4	5	6	7	8	9
0	0	4.313	8.598	12.83	16.98	21.01	24.92	28.69	32.29	35.71
10	38.96	42.04	44.89	47.56	50.05	52.35	54.47	56.44	58.23	59.86
20	61.36	-	-	-	-	-	-	-	-	-

TABLE 4

Values of R_3 for 75 - ohm T attenuator

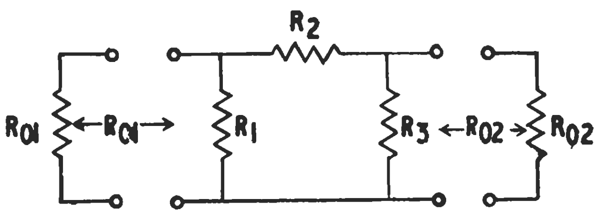
0.1 to 0.9 db

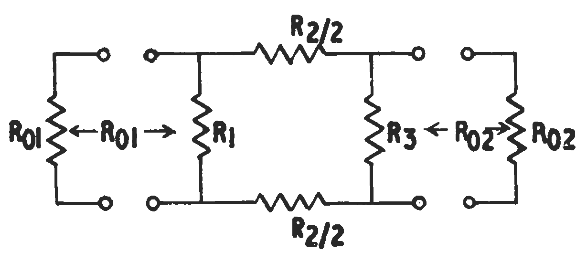
db	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
	6514.51	3247.4	2171.1	1624.3	1302.1	1084.9	929.6	813.1	722.5

1 to 20 db

db	0	1	2	3	4	5	6	7	8	9
0	∞	650	322.9	212.9	157.3	123.4	100.4	83.70	70.96	60.89
10	52.70	45.92	40.21	35.35	31.18	27.55	24.36	21.62	19.19	17.05
20	15.15	-	-	-	-	-	-	-	-	-

The π Attenuator





$$R_2 = \frac{(a^2 - 1)}{2a}\sqrt{R_{o1} R_{o2}}$$

$$\frac{1}{R_1} = \frac{1}{R_{o1}}\left(\frac{a^2 + 1}{a^2 - 1}\right) - \frac{1}{R_2}$$

$$\frac{1}{R_3} = \frac{1}{R_{o2}}\left(\frac{a^2 + 1}{a^2 - 1}\right) - \frac{1}{R_2}$$

For the symmetrical case where $R_{o1} = R_{o2} = R_o$

$$R_2 = R_o \frac{(a^2 - 1)}{2a}$$

$$R_1 = R_3 = R_o \left(\frac{a + 1}{a - 1}\right)$$

For 600 ohm attenuators the values of $R_1 + R_2$ are given in Tables 5 and 6.

TABLE 5

Values of $R_1 = R_3$ for 600 - ohm symmetrical π attenuator

0.1 to 0.9 db

db	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
	104240	52120	34747	26062	20852	17379	14898	13038	11591

1 to 40 db

db	0	1	2	3	4	5	6	7	8	9
0	∞	10435	5234	3509	2652	2142	1806	1569	1394	1260
10	1155	1071	1003	946.1	899.1	859.5	826.0	797.4	772.8	751.7
20	733.3	717.4	703.6	691.4	680.8	671.5	666.3	656.1	649.7	644.2
30	639.2	634.8	630.9	627.5	624.4	621.7	619.3	617.2	615.3	613.6
40	612.1	-	-	-	-	-	-	-	-	-

TABLE 6

Values of R_2 for 600 - ohm symmetrical π attenuator

0.1 to 0.9 db

db	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
	6.91	13.82	20.73	27.64	35.56	41.48	48.41	55.34	62.28

TABLE 6 (Contd.)

1 to 40 db

db	0	1	2	3	4	5	6	7	8	9
0	0	69.23	139.4	212.4	286.2	364.8	448.2	537.6	634.1	740.8
10	853.8	979.9	1119	1273	1444	1634	1845	2081	2345	2640
20	2970	3339	3753	4216	4736	5318	5971	6703	7524	8445
30	9499	10636	11936	13394	15030	16865	18924	21234	23826	26734
40	30000	-	-	-	-	-	-	-	-	-

The corresponding values of R_1 and R_2 for 75 ohm attenuators are given in Tables 7 and 8. For other values of R_o multiply values given in Tables 5 and 6 by $\frac{R_o}{600}$.

TABLE 7

Values of $R_1 = R_3$ for 75 - ohm π attenuator

0.1 to 0.9 db

db	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
	13030	6515	4343	3258	2607	2172	1862	1630	1449

1 to 20 db

db	0	1	2	3	4	5	6	7	8	9
0	∞	1304	653.0	438.8	331.5	267.8	225.8	196.1	174.2	157.5
10	144.4	133.9	125.4	118.3	112.4	107.9	103.3	99.68	96.60	93.96
20	91.66	-	-	-	-	-	-	-	-	-

TABLE 8

Values of R_2 for 75 - ohm π attenuator

0.1 to 0.9 db

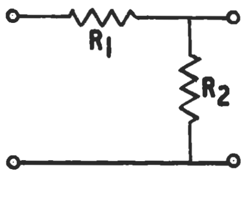
db	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
	0.864	1.728	2.591	3.455	4.445	5.182	6.051	6.918	7.785

TABLE 8 (Contd.)

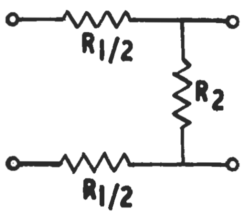
1 to 20 db

db	0	1	2	3	4	5	6	7	8	9
0	0	8.654	17.43	26.55	35.78	45.60	56.02	67.20	79.26	92.60
10	106.7	122.5	127.4	159.1	180.5	204.3	230.6	260.1	293.1	330.0
20	371.3	-	-	-	-	-	-	-	-	-

5. The L attenuator.



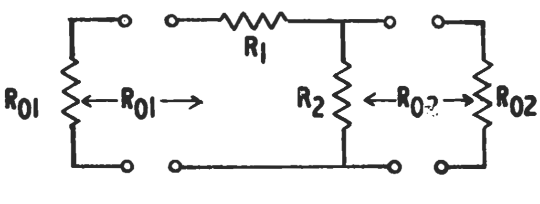
Unbalanced form



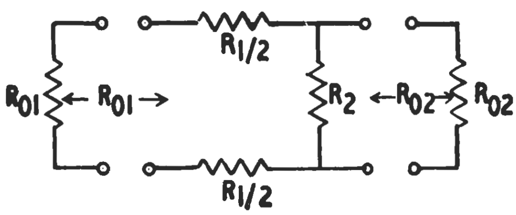
Balanced form

This type is simpler than either the T or π type, but simplicity is obtained at the expense of one degree of freedom in design. Thus the L attenuator can satisfy only two of the three conditions mentioned in par. 3 and for this reason it is not widely used. The following special cases are however of interest.

5.1 The minimum-loss matching pad. This provides image conditions at both ends, but the loss is dependent on the impedance-matching ratio. Where matching only is required this pad should be used in preference to either a T or a π pad.



Unbalanced form



Balanced form

Noting that Ro_1 must be greater than Ro_2.

$$R_1 = \sqrt{Ro_1 (Ro_1 - Ro_2)}$$

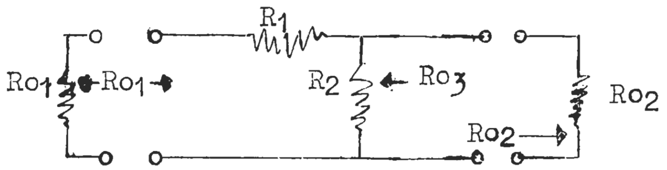
$$R_2 = Ro_2 \sqrt{\frac{Ro_1}{Ro_1 - Ro_2}}$$

Under such conditions the image attenuation will be given by

$$\text{Image attenuation} = 20 \log_{10} \left\{ \left[1 + \sqrt{1 - \frac{Ro_2}{Ro_1}} \right] \sqrt{\frac{Ro_1}{Ro_2}} \right\} \text{ db.}$$

5.2 L - attenuator providing image conditions at one end only and given insertion loss

5.2.1

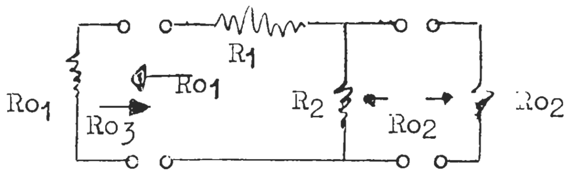


$$R_1 = Ro_1 \left[\frac{b\, Ro_1 + (b - 2)\, Ro_2}{b\, (Ro_1 + Ro_2)} \right]$$

$$R_2 = Ro_1 \left[\frac{2\, Ro_2}{b\, Ro_2 + (b - 2)\, Ro_1} \right]$$

and $$Ro_3 = \frac{R_2 (R_1 + Ro_1)}{R_1 + Ro_1 + R_2}$$

5.2.2



$$R_1 = \frac{b\, Ro_2 + (b - 2)\, Ro_1}{2}$$

$$R_2 = Ro_2 \left[\frac{b\, (Ro_2 + Ro_1)}{(b - 2)\, Ro_2 + b\, Ro_1} \right]$$

and $$Ro_3 = \frac{R_1 R_2 + R_1 Ro_2 + R_2 Ro_2}{R_2 + Ro_2}$$

An alternative use of the L-pad with image-matching at the output is to give a desired voltage ratio when a through-level measurement is to be made via a resistance probe by means of a low-impedance L.M.S. In this application put $Ro_1 = \frac{Ro}{2}$ and Ro_2 = Ro (where Ro is the impedance of the circuit where the level measurement is to be made and also the impedance of the L.M.S.). If the voltage-ratio desired is c (i.e. a correction of $20 \log_{10} c$ decibels is to be added to the L.M.S. reading) then R_1 and R_2 in the L-pad become:-

$$R_1 = \frac{c-1}{4} Ro \left[1 + \sqrt{1 + \frac{4}{c-1}} \right]$$

When c is large (error less than 1% for c>10) this relation becomes

$$R_1 = \frac{cRo}{2}$$

An alternative expression for R_1 in terms of Ro, c & R_2 is

$$R_1 = \frac{R_2 \, Ro \, (c-1)}{R_2 + Ro}$$

$$R_2 = \frac{Roc}{2c-3} \left[1 + \sqrt{1 + \frac{2c-3}{c^2}} \right]$$

When c is very large (error is 2% when c = 100) this relation becomes

$$R_2 = Ro$$

Typical values are as follows; the numbers in brackets are the actual values of R_1 and R_2 when Ro = 75.

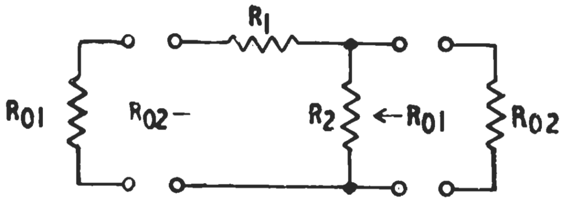
TABLE 9

Voltage-ratio, c	10	31.623	100
Correction, db	20	30	40
R_1	4.954 Ro (372)	15.796 Ro (1185)	50.00 Ro (3750)
R_2	1.225 Ro (90.7)	1.065 Ro (79.8)	1.020 Ro (76.5)
Tapping-loss, db	0.76	0.26	0.086

For other values of tapping loss, calculate the input resistance

$Ro_3 = R_1 + \frac{R_2 \, Ro}{R_2 + Ro}$ and refer to fig. 3 of the Technical Note "Transmission Losses" (Technical Note No. 40).

5.3 The Iterative - L Attenuator



NOTE: Ro_2 must be greater than Ro_1. There are three cases dependant upon which two quantities are specified.

5.3.1 Ro_1 and Insertion Loss specified.

$$R_1 = Ro_1 \; (b - 1)$$

$$R_2 = Ro_1 \; \frac{b}{(b - 1)}$$

and $Ro_2 = Ro_1 \, . \, b$

5.3.2 Ro_2 and Insertion Loss specified.

$$R_1 = Ro_2 \; \frac{(b - 1)}{b}$$

$$R_2 = Ro_2 \; \frac{1}{b - 1}$$

and $Ro_1 = Ro_2 \; \frac{1}{b}$

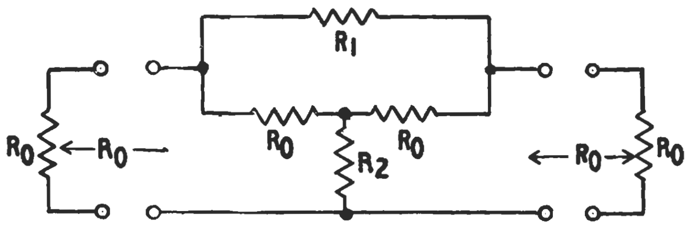
5.3.3 Ro_1 and Ro_2 specified.

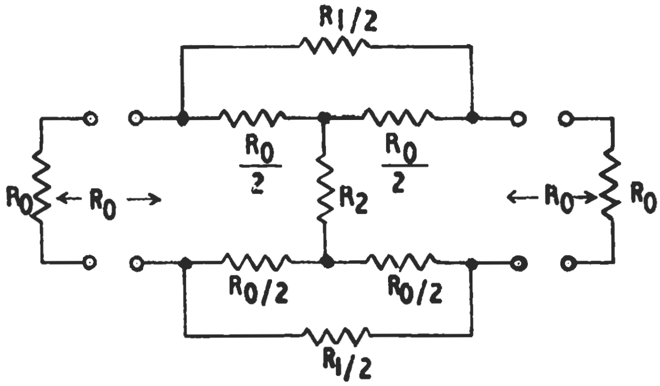
$$R_1 = Ro_2 - Ro_1$$

$$R_2 = \frac{Ro_1 \quad Ro_2}{Ro_2 - \; Ro_1}$$

and Insertion loss = $20 \log_{10} \frac{Ro_2}{Ro_1}$ db.

6. The Bridged - T Attenuator. The bridged T configuration is useful for switched variable attenuators as it requires only two variable elements in the unbalanced form, or three variable elements in the balanced form.





Unbalanced form

Balanced form

$$R_1 = Ro\ (a - 1)$$

$$R_2 = Ro\ \frac{1}{a - 1}$$

and notice $R_1\ R_2 = Ro^2$

Image attenuation $= 20 \log_{10} \sqrt{\frac{R_1\ R_2 + R_2}{R_2}}$

For 600 ohm attenuators the values of R_1 and R_2 are given in Table 10 and 11.

TABLE 10

Values of R_1 for 600 ohm Bridged T attenuator

0.1 to 0.9 db

db	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
	6.947	13.98	21.09	28.28	35.55	42.91	80.36	57.89	65.50

TABLE 10 (Contd.)

1 to 40 db

db	0	1	2	3	4	5	6	7	8	9
0	0	73.21	155.4	247.5	330.9	467.0	597.2	743.2	907.1	1091
10	1297	1529	1789	2080	2407	2774	3186	3648	4166	4748
20	5400	6132	6954	7875	8909	10070	11370	12830	14470	16310
30	18370	20690	23290	26200	29470	33140	37260	41880	47060	52880
40	59400	-	-	-	-	-	-	-	-	-

TABLE 11

Values of R_2 for 600 ohm Bridged-T Attenuator

0.1 to 0.9 db

db	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
	51820	25762	17075	12730	10130	8389	7149	6219	5496

1 to 40 db

db	0	1	2	3	4	5	6	7	8	9
0	∞	4917	2317	1454	1025	770.9	602.9	484.4	396.9	330.0
10	277.5	235.5	201.3	173.1	149.6	129.8	113.0	98.69	86.41	75.83
20	66.67	58.71	51.77	45.71	40.41	35.75	31.66	28.05	24.88	22.08
30	19.59	17.40	15.46	13.74	12.22	10.86	9.662	8.597	7.650	6.808
40	6.061	-	-	-	-	-	-	-	-	-

For other values of Ro multiply values given in Tables 10 and 11 by $\frac{Ro}{600}$.
Tables 12 and 13 give the corresponding values of R_1 and R_2 for Ro = 75.

TABLE 12

Values of R for 75 ohm Bridged-T attenuator

0.1 to 0.9 db

db	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
	0.868	1.747	2.636	3.535	4.444	5.364	6.295	7.236	8.188

TABLE 12 (Contd.)

1 to 20 db

db	0	1	2	3	4	5	6	7	8	9
0	0	9.151	19.42	30.94	43.87	58.38	74.6	92.90	113.4	136.4
10	162.1	191.1	223.6	260.0	300.9	346.8	398.5	456.0	520.8	593.5
20	675	-	-	-	-	-	-	-	-	-

TABLE 13

Values of R_2 for 75 ohm Bridged-T Attenuator

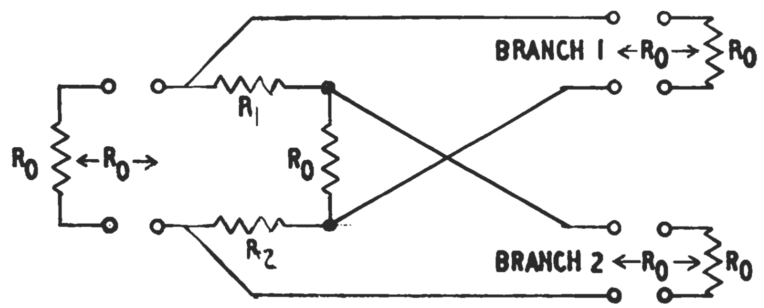
0.1 to 0.9 db

db	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
∞	6478	3219	2131	1716	1266	1049	893.6	777.4	687.0

1 to 20 db

db	0	1	2	3	4	5	6	7	8	9
0		6148	289.6	181.8	128.1	96.36	75.36	60.55	49.61	41.25
10	34.69	29.44	25.16	21.63	18.70	16.23	14.13	12.34	10.80	9.479
20	8.334	-	-	-	-	-	-	-	-	-

7. The Spur Attenuator or Resistance Hybrid. This attenuator is of use where it is necessary to branch a unidirectional circuit and has the advantage that there is no interaction between the branches.



$R_1 = Ro\ k$; $R_2 = Ro\ \frac{1}{k}$

By choice of value for the ratio k the ratio in which the power is divided between the branches can be varied as shown in Table 14.

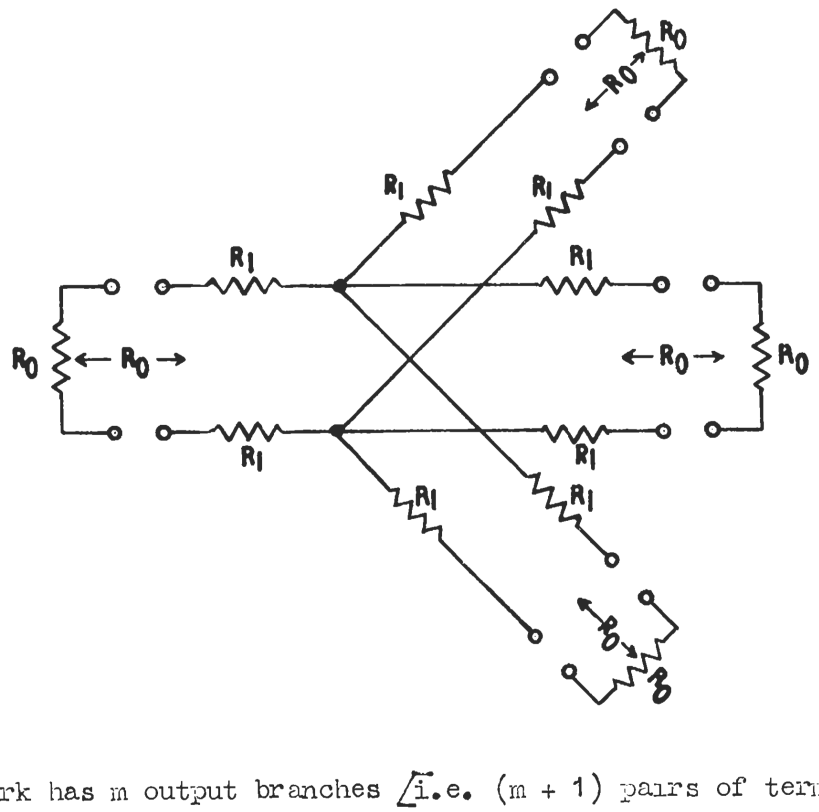
TABLE 14

Resistance Hybrid

Relationship between k $(=\sqrt{\frac{R_1}{R_2}})$ and losses between Input and the Output branches

k	Insertion loss between input and branch 1 (db)	Insertion loss between input and branch 2 (db)
1	6.0	6.0
2	3.5	9.5
3	2.5	12.0
4	2.0	14.0
10	1.0	21.0
30	0.28	30

8. Branching Networks. These networks can be used for dividing a circuit into any number of branches, whilst presenting image conditions at each branch. There is no isolation between branches so that a mis-match or mis-termination at one branch would affect all other branches.



If the network has m output branches [i.e. (m + 1) pairs of terminals]

$$\text{then } R1 = \frac{Ro}{2} \quad \frac{(m - 1)}{(m + 1)}$$

Image attenuation = 20 $\log_{10}$ m db.

Table 15 gives values of R_1 and image attenuation for various values of m, Ro being 600 ohms.

TABLE 15

Values of R and Image attenuation for 600 ohm Branching Network

m	R_1 ohms	Image attenuation of network common input to any branch output (db)
1	0	0
2	100	6
3	150	9.5
4	180	12.0
5	200	14.0
6	214.3	15.6
7	225	16.9
8	233.3	18.1
9	240	19.1
10	245.4	20.0
11	250	20.8
12	253.8	21.6

9. Component Tolerances. For the particular cases of very small or very large attenuations, simple expressions can be derived for the symmetrical T or π attenuator giving the change in image attenuation produced by small variations from the nominal value of series or shunt resistors. The asymptotic expressions, accurate for extreme values, have been plotted as straight lines in the chart on p11 and connected by transition curves. Separate graphs have been drawn for resistor tolerances of 1%, 2%, 5% and 10% and give the maximum total change in image attenuation when shunt and series elements of T or π attenuators simultaneously diverge by the stated percentage from the nominal value. As an attenuator is normally used between fixed terminations an additional loss would be introduced by the change in image impedance from the nominal value of the terminations. For the small changes considered this will be negligible and the graphs may be used to show changes in insertion loss.

For attenuation of less than 5 db.

Percentage error in db = Percentage error in R_1.

$$\text{Percentage error in Ro} = \frac{(\text{Percentage error in } R_1 + \text{Percentage error in } R_3)}{2}$$

For attenuation above 40 db

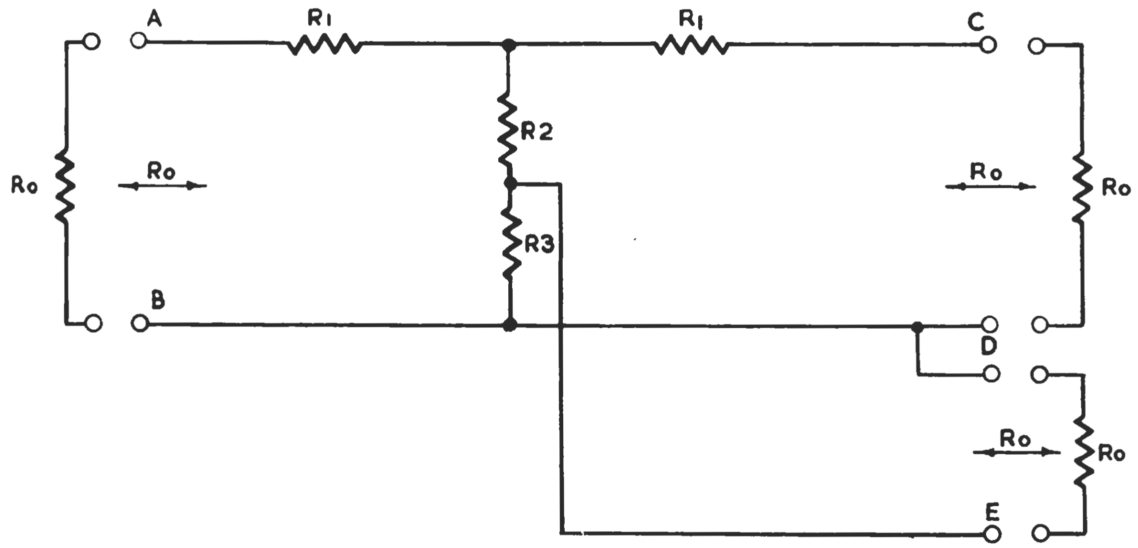
Attenuation error in db = 8.6 (fractional error in R_1 - fractional error in R_3)

Percentage error in Ro = Percentage error in R_1.

10. Monitor Pads

10.1 These networks are inserted in a transmission path, usually to provide a low-level monitoring point whilst introducing only little loss in the main path. Image-terminating conditions apply at each of the three pairs of terminals.

If the loss introduced in the main through path AB to CD is 20 $\log_{10}$ A db and the loss via the branch monitor path AB to ED is 20 $\log_{10}$ M db,



Then given M; A may be calculated from

$$A = \frac{M^2}{M^2 - 2M + 2} \quad \text{.......(1)}$$

$$\text{and } R_1 = \left(\frac{M - 1}{M^2 - M + 1}\right) Ro \quad \text{.......(2)}$$

$$R_2 = \left(\frac{M (M - 1)^2}{2 (M^2 - M + 1)}\right) Ro \quad \text{.......(3)}$$

$$R_3 = \left(\frac{M}{M - 2}\right) Ro \quad \text{.......(4)}$$

10.2 If a maximum permissible value for A is stated, M may be calculated from

$$M = \frac{A + \sqrt{2A - A^2}}{A - 1}$$

and $$R_1 = \frac{A - 1}{A + 1} Ro$$

$$R_2 = \frac{Ro\,(Ro + R_1 + R_3) - R_1 R_3}{2\,(R_3 - Ro)}$$

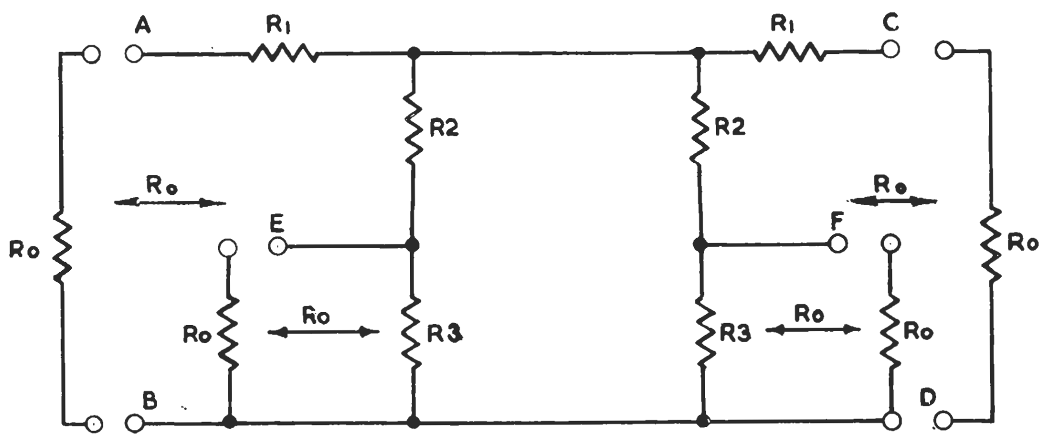
$$R_3 = \sqrt{\frac{A}{2 - A}}\; Ro$$

It should be noted that with the circuit configuration shown the maximum value of A is 2 (6 dbs) and the minimum value for M is 2.

11. Double Monitor Pads

11.1 These networks are inserted in a transmission path when two low-level monitor points are required with only little loss in the main path. Image terminating conditions apply at each of the four pairs of terminals.

The loss introduced in the path AB to CD is $20 \log_{10} A$ db and the loss via either of the monitor path AB to ED or AB to FD is $20 \log_{10} M$ db.



$$R_1 = \left(\frac{A - 1}{A + 1}\right) Ro$$

$$R_2 = \left(\frac{4A}{A^2 - 1} - \frac{2\sqrt{A}}{2\sqrt{A} + \sqrt{3 + 2A - A^2}}\right) Ro$$

$$R_3 = Ro\sqrt{\frac{4A}{3 + 2A - A^2}}$$

$$M = \frac{A + 1}{2}\left(1 + \frac{R_2}{Ro} + \frac{R_2}{R_3}\right) = \frac{Ro}{Ro - R_1}\left(1 + \frac{R_2}{Ro} + \frac{R_2}{R_3}\right)$$

$$A = \frac{Ro + R_1}{Ro - R_1}$$

It should be noted that with the circuit configuration as shown A must not exceed 3 (or be less than 1). A and M cannot be expressed simply in terms of each other without R_1, R_2 or R_3 appearing in the expressions.

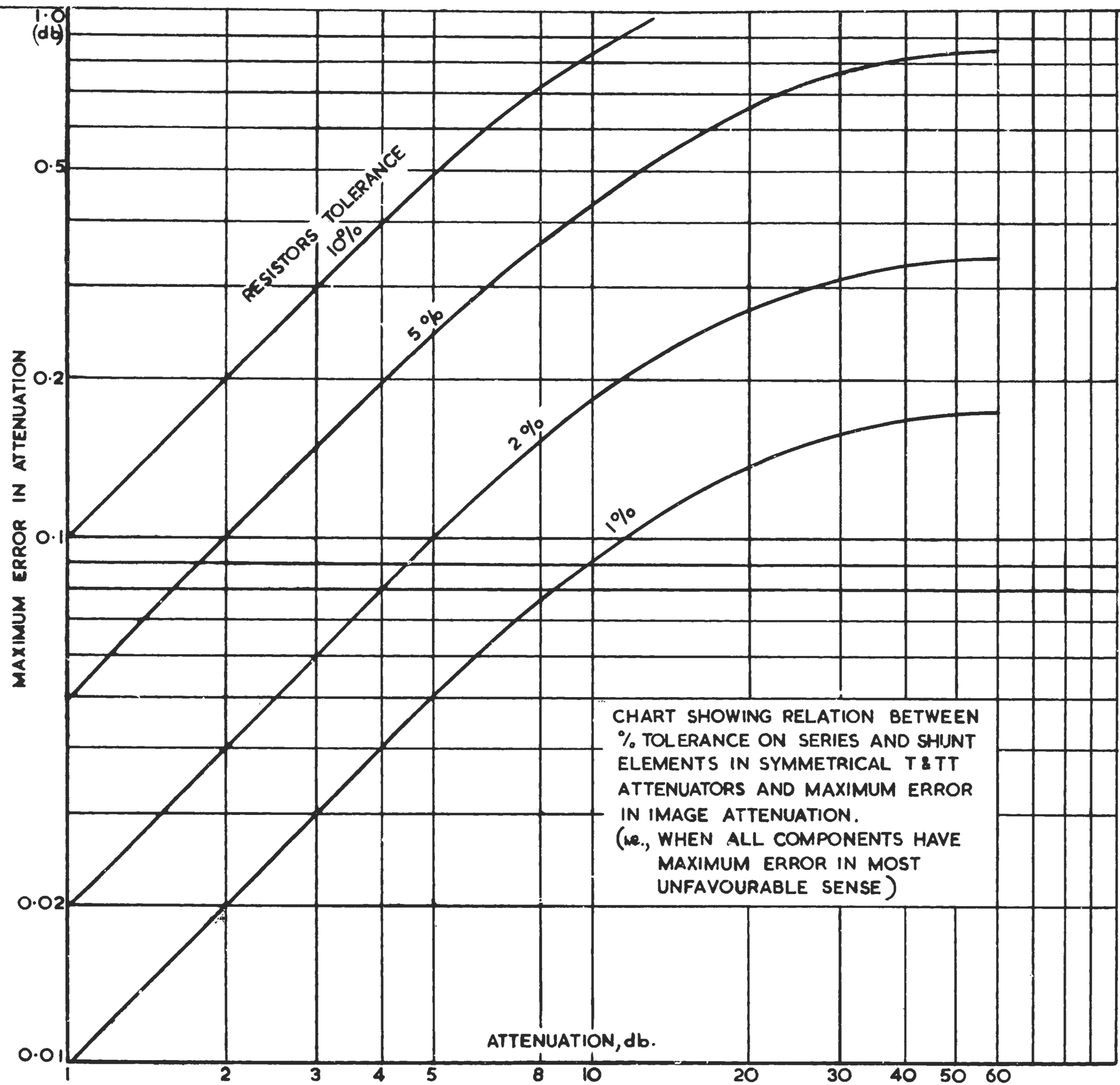


CHART SHOWING RELATION BETWEEN % TOLERANCE ON SERIES AND SHUNT ELEMENTS IN SYMMETRICAL T & TT ATTENUATORS AND MAXIMUM ERROR IN IMAGE ATTENUATION.
(i.e., WHEN ALL COMPONENTS HAVE MAXIMUM ERROR IN MOST UNFAVOURABLE SENSE)