

**ENGINEERING TRAINING
SUPPLEMENT**

No. 6

PROGRAMME METERS

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PROGRAMME METERS

by

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PROGRAMME METERS

INTRODUCTION

The purpose of this supplement is to give engineers a clear picture of the part played by programme meters in the broadcasting chain, and to explain the development of circuits associated with peak programme meters in current use. The function of the programme meter is to provide visible indication of variations in programme volume ; these variations must be confined to maximum and minimum values conditioned by the limits of intensity which the transmission system can handle. But let us begin the story at the beginning.

The broadcasting chain is unfortunately limited in the range of sound intensity which it can handle and some provision must be made to ensure that these limits are not over-stepped. At first sight it would not appear that this is a serious problem, but in fact there are some considerable difficulties to be overcome. These difficulties arise from two fundamental causes. In the first place the ear which is, after all, the final subjective measuring device, has a logarithmic characteristic with respect to intensity. This means that for a given increase in sensitivity the ear experiences an increase in loudness equal to the logarithm of the increase in sensitivity. (See Recording Training Manual, page 4.) Thus a fourfold increase in sensitivity approximately doubles the sensation of loudness. Moreover, the ear is capable only of assessing absolute intensity in very general terms and judges loudness largely by comparison. On the other hand transmission systems in general have a linear characteristic with well-defined maximum and minimum limits, the maximum being the power handling capacity and the minimum the background noise inherent in the system. Unfortunately these limits usually delineate an intensity range which is much less than that normally experienced by the ear in every-day life. It is therefore necessary to compress the intensity range applied to the transmission system.

The other difficulty arises from the fact that the impression of loudness is not due only to the peak sound pressure or intensity ; it is determined by the energy in the sound wave which is in turn a function both of the amplitude of the wave and of its shape. Some sounds, of which speech and percussion instruments provide good examples, contain a large number of short impulsive waves whose peak value is large and duration short (see Fig. 1a). Other sounds, such as those produced by the stringed and wind instruments

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of the orchestra, have wave shapes which are much less “spiky” in appearance, (see Fig. 1b). In consequence the peak value is much less in a wave of the latter type, although the two sounds illustrated in Fig. 1 may have the same apparent loudness.

The conflicting aspects of the problem now become apparent. On the one hand it is necessary to compress the intensity range (for the reason already explained) in such a way that the apparent values of loudness of the various forms of programme are in approximately the same relationship to each other as in the original sound ; on the other hand it is desired to make the fullest economic use of the transmission system, which in broadcasting

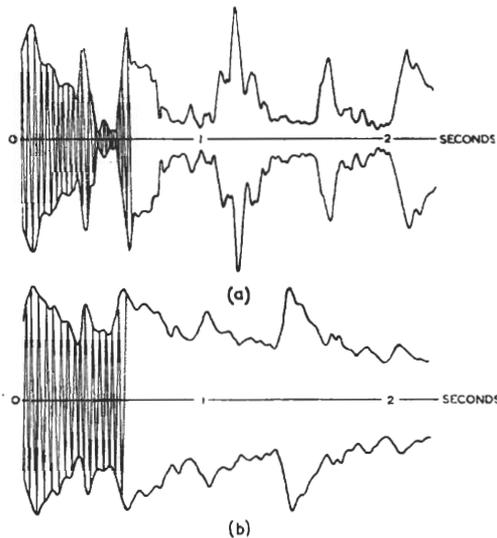


FIG. 1. COMPARISON OF SOUND WAVEFORMS

- (a) Shows the peaky form of speech and percussion instruments
- (b) Shows the smoother form of string and wind instruments

means that the fullest possible modulation depth of the most costly item, the transmitter, should be used at all times. Since this modulation depth bears only an indirect relationship to the loudness of sounds transmitted, we are faced with a problem which can only be resolved by compromise.

Programme Meters

Let us return for a moment to the remarks made about the capabilities of the ear. It will be clear that since the ear cannot accurately assess

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absolute levels, it is necessary to resort to some other system of measurement, no matter whether it be intensity or loudness, if any accurate indication of either of these quantities is to be obtained. Visual indication is the obvious choice and so the Programme Meter comes into the picture.

A Programme Meter differs from any ordinary meter in that it is required to indicate a quantity which is rapidly changing and here another arbitrary factor arises. Variations of both peak intensity and of loudness occur at a rate which is faster than the eye can follow ; moreover, the impression of loudness itself is a subjective effect based on the integration of sound energy over some unspecified time. To give a visual indication, therefore, of either peak intensity or loudness it is necessary to provide the indicator with some slow-acting or integrating characteristic which in the final analysis can only be arbitrary.

In view of what has been said, therefore, it is quite understandable that two general classes of Programme Meter have established themselves ; indeed it is perhaps surprising that there are not more. On the one hand the Peak Programme Meter broadly measures the peak value of the programme and enables a watch to be kept on the modulation amplitude handled by the system at the expense of the loudness indication. On the other hand the Programme Volume Meter measures the energy in the sound waves and gives a more reliable indication of loudness ; its disadvantage is that it introduces a tendency to occasional overload and distortion, which can only be avoided by reducing the depth of modulation, or by increasing the power-handling capacity of the transmitter. On this basis, the advantages of the programme volume meter are to some extent offset by a somewhat uneconomic use of the transmission system. The former class of meter is in general use in European equipment and the latter in American equipment, although there are many exceptions to this demarkation.

Units of Measurement

Because programme amplitude is a constantly varying quantity, whose value is arbitrarily recorded by a meter with certain specified characteristics, some terms must be adopted to designate the unit which the peak programme meter or programme volume meter registers. The intensity of steady tone is taken as a basis and is measured as a voltage relative to that corresponding to 1 mW in 600 ohms. The level of any other continuous tone is expressed in decibel units above or below this value and these units in American practice are designated " dbm " ; it should be clearly understood that dbm units can only be applied to steady tone.

When the Programme Volume Meter is being used the unit adopted to designate the meter reading on programme is the Volume Unit or V.U. and it is defined as the level of programme which causes the Volume Meter to kick frequently to its 100 per cent mark (see below). Thus a programme intensity of + 10 V.U.s is said to exist when the V.U. meter calibrated to read 100 per cent on tone of + 10 dbm kicks to the 100 per cent mark. In BBC practice the corresponding term is " Programme Volume " which is

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defined as the level of programme which causes the BBC programme meter to peak to "6" (the present equivalent to 100 per cent on the V.U. meter) when the meter has been calibrated to read "4" on the corresponding steady tone. Tone amplitude is always referred to in the BBC as "level"; it is important to notice that this again is a term which can only be applied to the steady state.

Volume Meters

The simplest volume meter is undoubtedly the American Volume Unit (or V.U.) Meter which consists of a meter with special characteristics, together with a dry rectifier and a ballast resistance. This combination in series may be merely shunted across a 600-ohm line on which the programme volume is to be measured. It will be clear that the meter must derive all the

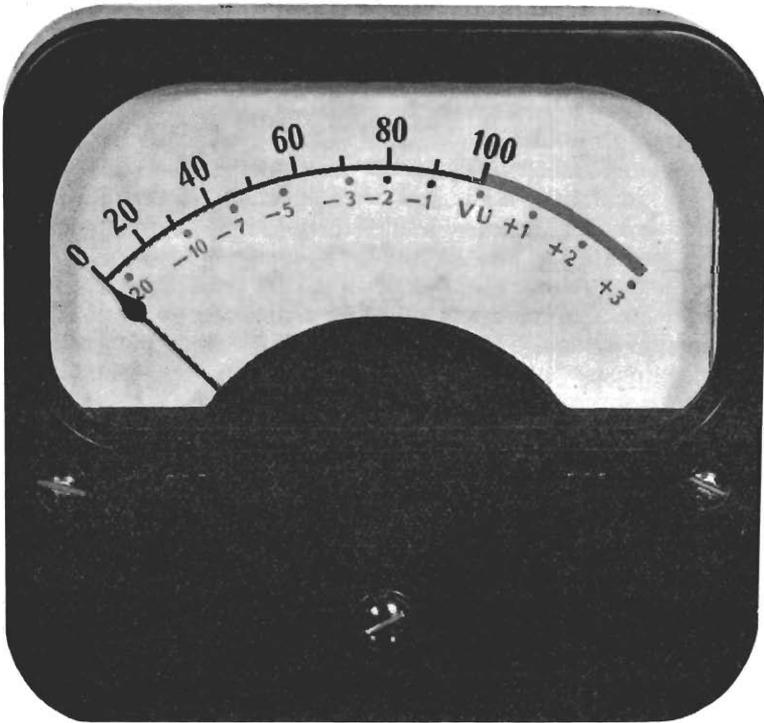


FIG. 2. VOLUME METER, SHOWING TYPICAL SCALES

energy necessary to operate it direct from the programme currents, and since these are intermittent the meter reading will be proportional to the energy in the wave and therefore to the loudness of the sound it represents. Fig. 2 shows the scale of this type of meter.

The more impulsive the sound, however, the greater will be the peak

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value relative to the meter reading compared with their relative values on steady tone. When lining up the system, therefore, account has to be taken of this fact and the modulation depth of the transmitter set well below 100 per cent when the meter is giving full-scale reading on steady tone. As a result of trial and error it is generally assumed that a factor of about 10 db gives a reasonable margin of safety in this respect so that a modulation depth of the order of 35 per cent is usually adopted when the maximum level of steady tone is being indicated. This figure, however, is quite arbitrary and varies from one organisation to another.

The reading of the V.U. meter for a given impulsive sound will, of course, depend on the sensitivity of the meter, on its ballistic characteristics and also upon the rectifier characteristic. These features have therefore been standardised* and the performance specification is briefly as follows.

- (i) The rectifier characteristic shall be of the form $i = ke^{1.2}$ (e being the applied e.m.f., i the current passed and k is a constant).
- (ii) The meter is scaled so that the 100 per cent. mark is at approximately 0.71 of full-scale deflection.
- (iii) The impedance of the meter and rectifier when indicating 100 per cent under the influence of an applied sinusoidal signal is to be 3,900 ohms.
- (iv) The sensitivity of the combination is to be such that when an e.m.f. of 1.228 volts r.m.s. is applied in series with an additional resistance of 3,600 ohms (making 7,500 ohms in all) the deflection will lie between the 100 per cent mark and 0.8 of full scale. (In use an additional variable resistance is included in the circuit and adjusted on line-up to give the precise required sensitivity.)
- (v) The speed of response to be such that if the above signal is suddenly applied the needle shall read 99 per cent of its final deflection in 0.3 seconds and shall overswing by not less than 1 per cent and not more than 1.5 per cent.

It will be noticed from (iii) and (iv) that the shunting impedance of the standard meter combination is approximately 7,500 ohms and that it gives 100 per cent reading when a tone level of + 4 dbm exists across a 600-ohm line. Reduction of the external ballast resistance would increase the sensitivity at the expense of greater shunting loss, but this is undesirable because the variation of the rectifier impedance with change of applied voltage is quite large and unless adequately swamped causes distortion of the waveform on the line.

BBC Practice

In the very early days of broadcasting the BBC used a slide-back valve voltmeter for indicating when the peak value of the programme transmitted exceeded the permitted maximum, but this was soon replaced by a ballistic meter somewhat similar to the V.U. meter described above. Instead of the

* Howard A. Chinn, "The Measurement of Audio Volume," *Audio Engineering*, 1951, Vol. 35, No. 9, No. 10.

PROGRAMME METERS : BBC PRACTICE

dry rectifier, however, various valve circuits were used to feed it, these circuits being so arranged that not only was rectification obtained but also the mean current through the meter was approximately a logarithmic function of the r.m.s. grid input voltage. In this way a programme meter was obtained which had an approximately logarithmic scale with good sensitivity and low shunting loss. Both these features represent a considerable advantage over the V.U. meter, at the expense of the necessary valve unit and its power supplies.

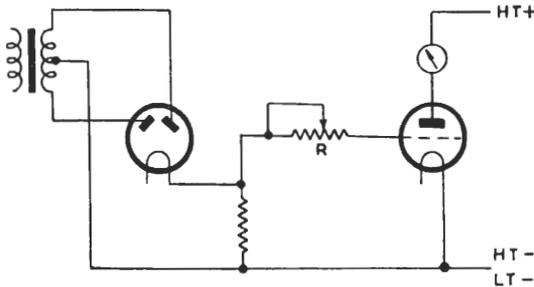


FIG. 3. TYPICAL PROGRAMME-METER CIRCUIT : GRID-CURRENT OPERATION

Two typical circuits used are shown in Figs. 3 and 4. Fig. 3 shows the grid current type in which rectification was produced by a double-wave diode rectifier and the logarithmic law was obtained by a combination of the effects of the anode-current/grid-volts and the grid-current/grid-volts characteristics of a triode valve. Adjustment of resistance R gives some control over the law. Fig. 4 shows the pentode type in which two pentodes

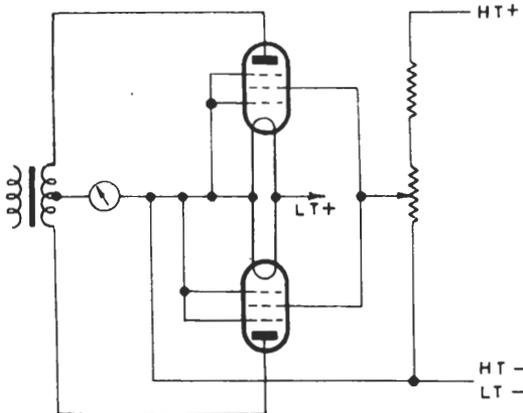


FIG. 4. TYPICAL PROGRAMME-METER CIRCUIT : DOUBLE PENTODE OPERATION

in push-pull were employed both as the rectifier and also to give the logarithmic characteristic. In this case the anode/cathode circuit was used for rectification (diode-fashion) and adjustment of the screen potential gave

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some control of law. Both circuits have been simplified in this description and were normally preceded by an amplifier stage (or stages).

About 1936 it was realised that the ballistic meter was a somewhat hit-and-miss method of trying to load fully the transmission system ; this loading, of course, affects the service area of a transmitter. For example, tests showed that, compared with steady tone, programme controlled to give the same peak readings on the meter gave actual peaks of modulation varying from 2 or 3 db higher on very legato music to 15 or 16 db higher on German speech. The setting of the line-up level at 8 to 10 db below 100 per cent modulation therefore was as good a compromise as could be hoped for but nevertheless a compromise. Accordingly it was decided to develop a very quick acting peak-reading voltmeter with a logarithmic scale.

Peak Programme Meter

Initially a circuit was developed which was capable of fully registering peaks lasting only 10 microseconds but this speed of response was found quite unnecessary. Extended tests were carried out with a unit in which 80 per cent. of the full peak value of a square wave was registered in 0.5 milliseconds, from which the interesting fact emerged that if the duration of a peak is very short the ear does not get time to detect distortion produced

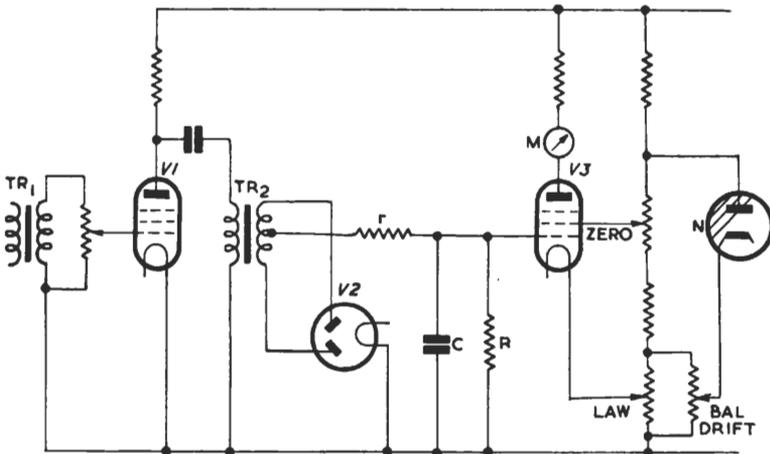


FIG. 5. BASIC PROGRAMME-METER CIRCUIT AS FINALLY DEVELOPED

in the transmission system by momentary over-modulation. Therefore in general the shorter the peak the greater the percentage of overmodulation that can be tolerated ; further tests showed that a circuit registering 80 per cent of the peak value of a square wave in 4 milliseconds provided the best compromise. Such a meter will not record fully peaks of shorter duration and will therefore cause the operator to control his programme in such a way that momentary overloads will take place ; but the resulting distortion will, by and large, pass unnoticed by the ear and the average modulation depth will be correspondingly raised.

PEAK PROGRAMME METER

As finally developed, the peak programme meter amplifier consists basically of the circuit shown in Fig. 5. V1 is a buffer valve whose function is to give the required sensitivity and to isolate the variations of rectifier impedance from the feeding circuit. TR2, V2, C, r and R constitute a full-wave peak rectifier circuit which is the essence of the unit. The capacitor C charges to the peak value of the signal on either half of the secondary winding of TR2 but the rapidity with which it reaches this value is determined by the time constant of the capacitor and the charging resistance formed by r , the effective forward diode resistance and the output impedance of V1. The value of 2.5 milliseconds which has been adopted for this time constant corresponds to a rate of rise to 80 per cent in 4 milliseconds as mentioned earlier. (Note that a full-wave rectifier is employed because the positive and negative wave peaks on some sounds, particularly speech, may differ by as much as 8 db.)

The rapidity with which the capacitor C loses its charge when the signal is removed is determined only by the resistance R and is set to a value which allows the meter needle to fall back quickly enough to register the next important peak but not so quickly as to cause eye-strain or to have any effect on the charging characteristic. A time constant ($R \times C$) of 1 second has been set for this as compared with the time constant of 2.5 milliseconds adopted for the charging circuit. Common values are 0.2 μ F for C, 5 megohms for R and 12,500 ohms for the total charging impedance which is adjusted by variation of r on test as mentioned later ; these values vary somewhat in different units, but their relative values are always the same.

V3 is a variable- μ pentode valve which is biased so that the change of anode current is approximately a logarithmic function of the variation of potential between grid and earth. This relationship can only be obtained by applying low bias to V3 with no signal present and causing C to be charged so that additional negative bias is applied to V2. Thus the anode current is a maximum for zero applied signal and the meter reading must be backed off in some way. This is done by using a meter with a right-hand zero which therefore takes up the left-hand position on no signal and moves again to the right as the signal increases and the anode current is cut down.

If a very long discharge time-constant had been chosen for C and R any normal form of meter would have been adequate to register the voltage reached by C as the result of a short signal peak before the decay became appreciable. A decay time-constant of 1 second, however, means that the voltage of C decays at the rate of 8.7 db per second or nearly 1 db in 1/10th second. It will be realised therefore that the time taken for full scale registration by the meter must be at least as short as this if an accuracy within 1 db is to be obtained. A specially quick-acting meter is therefore required and the specification calls for a meter to satisfy the following tests.

(i) When a voltage V , which if steadily applied to the meter in series with 100 kilohms gives a full scale deflection, is suddenly applied to 10 μ F, 100 kilohms and the meter in series, the meter needle shall reach 97 per cent f.s.d. (full-scale deflection).

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(ii) When this voltage V is suddenly applied to 100 kilohms and the meter only in series, the meter needle shall not overshoot f.s.d. by more than 5 per cent.

The only remaining features of the circuit to be noted are that adjustments of the cathode and screen-feed potentiometers set the standing anode current (and hence the meter zero) to a fixed datum. This datum current can be obtained with an infinite number of combinations of these two adjustments, but each combination will give a slightly different law to the valve anode-current/grid-volts characteristic. By judicious adjustment of these two controls therefore (together with the sensitivity control) it is possible to compensate for variations of characteristic between valve and valve and to line up the majority of valves with the fixed law of the meter scale.

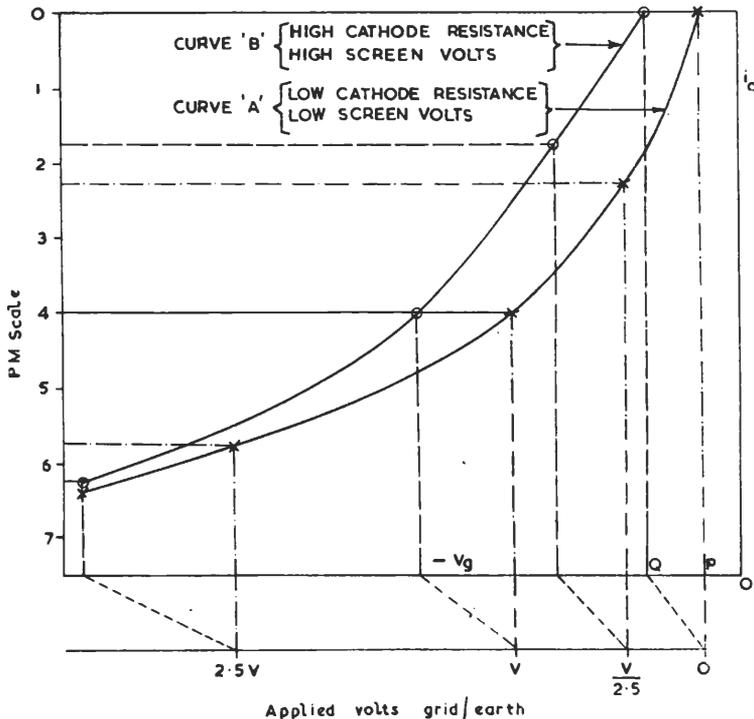


FIG. 6. CURVES ILLUSTRATING ADJUSTMENT OF PROGRAMME METER "LAW"

Fig. 6 shows how this adjustment of law comes about. Curve A is the characteristic obtained with a low value of bias (P) and the datum current adjusted by the screen potential (to give the P.P.M. zero reading). An additional bias of V due to a steady signal reduces the current to the value "4" (the sensitivity of the unit and therefore the value of V being adjusted till this is so). Increase or decrease of V by 8 db to approximately $2.5V$ and $V/2.5$ respectively gives P.P.M. readings below "6" and above "2" respectively (i.e., the scale requires opening). Curve B now shows the characteristic

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obtained with a larger value of initial bias (Q); note particularly the straightening of the curve at high anode current due to the cathode resistance feedback effect. Again the datum current is adjusted at the minimum bias, Q . Again a signal giving a voltage V is applied and the value of V adjusted till "4" is obtained on the P.P.M. (see dashed line, Fig. 6). It will now be seen that increase or decrease of V by 8 db gives P.P.M. readings above "6" and below "2" respectively (i.e., the scale is now too open).

The datum current depends to some extent on supply voltages and other factors and if this is corrected by the screen volts adjustment the minimum change of characteristic occurs. Accordingly this adjustment is now used as the zero control. In order to reduce the effect of variations of supply as much as possible the neon lamp N is used to stabilise the screen voltage against variations of h.t. potential and by adjustment of the potentiometer to which it is connected, considerable compensation for zero drift due to the effect of mains supply variations can be obtained.

Testing

The method of adjusting the law, setting zero, adjusting drift compensation and lining up is given in detail in Technical Instruction S4, Section H, and would be out of place here. It is pertinent, however, to discuss the overall test for operating speed. To take the simplest test first, the decay time constant of 1 second represents a fall in meter reading of 26 db in 3 seconds. 26 db is the interval between the marks "1" and "7" on the meter scale (see below) and a simple stop watch check when a tone which deflects the meter to "7" is cut is all that is required. The tolerance is ± 0.5 seconds.

In order to check the charging speed of the instrument a standard pulse is employed. The simplest available standard pulse which is easily reproduced is the discharge of a known capacitor (charged to a known voltage) through a known resistance. It is shown in the appendix that if V_0 is the peak value of a tone which registers "7" on the P.P.M. then a capacitor of $5 \mu\text{F}$ charged to a potential V_0 and discharged through 600 ohms will cause the meter to peak to "5"; this is the standard test for operating speed with a tolerance of ± 1 db and any correction necessary is obtained by adjustment of the resistance r in Fig. 5.

It is of interest to note that in order to pass this pulse on to the diode integrating circuit substantially undistorted, an amplifier with exceptionally good low-frequency response is required; some modifications of certain units incorporating peak programme meters had to be made at one time to ensure that this was the case. This pulse is more searching in this respect than is ordinary programme material.

Miscellaneous

One of the advantages of the P.P.M. instrument over the V.U. meter is that for prolonged observation the slower movement of the former results in considerably less eye-strain. In order to reduce this strain as much as possible

PROGRAMME METER : MISCELLANEOUS

a black scale with white pointer and lettering has been adopted as shown in Fig 7. The number of marks on the scale was reduced to the minimum for the same reason and intercepts of 4 db only are shown. These line up well with the usual 2 db per step mixer and gain controls used in the Corporation,

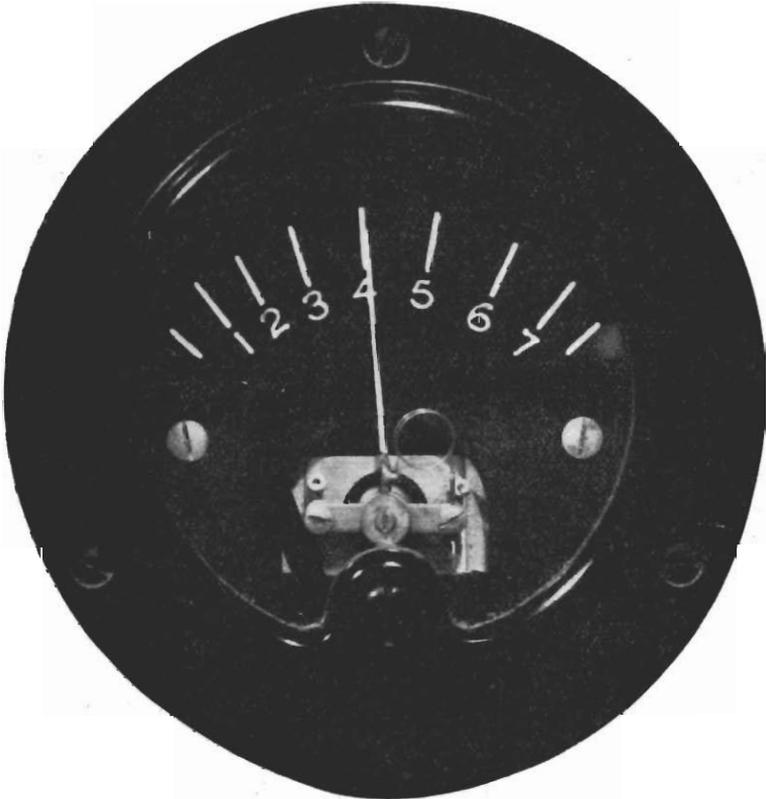


FIG. 7. PROGRAMME METER CALIBRATED IN 4-DB STEPS
This is the BBC standard meter

and purely arbitrary figures have been assigned to them. It was at one time customary to adjust the sensitivity of the meter so that normal line-up tone registered "5." Present practice, however, is to line up the meter on a standard tone so that "4" is registered when the tone level corresponds to 40 per cent modulation. A peak to "6" therefore represents 100 per cent modulation and the range from "6" to "7" indicates the extent that any overload has inadvertently been allowed to exceed this limiting level. This precise indication is found to be very valuable in operational control. Normal practice in the BBC also is to adjust quiet passages so that peaks below "2" do not persist for long. It should be noted that for the intercept on the scale between "1" and "2" the indicated level difference is 6 db and not 4 db as for the other intercepts.

PROGRAMME METER : SLUGGED RESPONSE

The basic P.P.M. circuit is employed in a variety of amplifier and programme units. Examples may be quoted

O.B. Amplifier OBA/8
Amplifier APM/1
Recording Amplifier DR/5A
Peak Programme Meter PPM/2
Monitoring Amplifier MNA/1
Portable Peak Programme Meter PPM/6
Test Peak Programme Meter TPM/3

In some of these units the buffer valve V1 shown in the circuit of Fig. 4 may also perform other functions, e.g., in the OBA/8 it is in fact the output valve. Precautions are taken to see that distortion due to variations of impedance of the diode circuit during portions of the wave cycle are not reflected into the programme output.

Slugged Response

It has been found in practice that when it is necessary to check programme volumes between two points on the S.B. system it is extremely difficult for two operators to compare the fluctuating readings of their instruments. An attempt to overcome this trouble was first made by enabling the diode integrating capacitor in the P.P.M. Circuit to be increased in value thereby slowing up the movement of the meter in both directions. This did not give satisfactory results in practice so that a different method is now used.

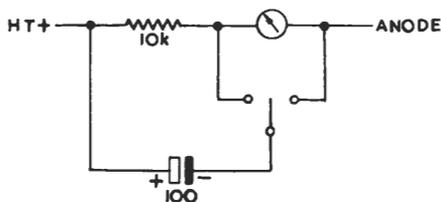


FIG. 8. CIRCUIT FOR PRODUCING SLUGGED RESPONSE

A resistance of 10 kilohms is connected in series with the meter itself and a capacitor of $100 \mu\text{F}$ is connected across the combination. The arrangement is shown in Fig. 8. The slugging effect on the meter thus obtained is found to give a satisfactory indication of the average programme volume which can be compared with accuracy with an adjacent station on the S.B. circuit.

When the slugging effect is not required the capacitor is switched across the resistance only, thereby maintaining the polarising potential yet removing its influence on the speed of the meter. In this connection it should be remembered that the meter circuit is fed from a pentode anode circuit which is of high impedance so that additional moderate impedances in series in the circuit have little effect.

APPENDIX

Calculations to find the Effect of a Test Pulse

The expression for the voltage during the discharge of the test circuit, CR, Fig. 9 is

$$V = V_0 e^{-t/RC}$$

The instantaneous conditions in the P.P.M. circuit are therefore

$$\begin{aligned} ri + \frac{Q}{k} &= V \\ &= V_0 e^{-t/RC} \end{aligned}$$

where r is the charging resistance, k the integrating capacitance, Q the charge in the capacitor and $V_k < V$.

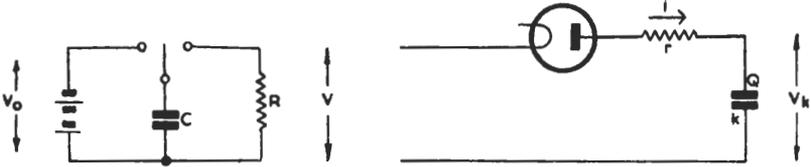


FIG. 9. PROGRAMME METER TEST CIRCUIT

Take the Laplace transforms of both sides, i.e., multiply through by ϵ^{-pt} , and then integrate with respect to t from 0 to ∞ .

$$\text{Let } \bar{i} = \text{Laplace transform of } i = \left(\int_0^{\infty} i \epsilon^{-pt} dt \right)$$

$$\text{Let } \bar{V} = \text{Laplace transform of } V$$

$$\therefore r\bar{i} + \frac{1}{k} \int_0^{\infty} \epsilon^{-pt} Q dt = \bar{V}$$

$$\therefore r\bar{i} - \frac{1}{pk} \left[\epsilon^{-pt} Q \right]_0^{\infty} + \frac{1}{pk} \int_0^{\infty} \epsilon^{-pt} \frac{dQ}{dt} dt = \bar{V}$$

$$\text{Since } i = \frac{dQ}{dt}$$

$$r\bar{i} + \frac{Q_0}{pk} + \frac{\bar{i}}{pk} = \bar{V}$$

APPENDIX

Under the conditions of the test, $Q_0 = 0$ (i.e., the initial charge on k is zero).

$$\begin{aligned} \therefore \bar{i} \left(r + \frac{1}{pk} \right) &= V_0 \int_0^{\infty} \epsilon^{-t/RC} \epsilon^{-pt} dt \\ &= - \frac{V_0}{p + 1/RC} \left[\epsilon^{-t(p + 1/RC)} \right]_0^{\infty} \\ &\quad \text{Since } \epsilon^{-\infty} = 0 \text{ and } \epsilon^{-0} = 1 \\ &= \frac{V_0}{p + 1/RC} \end{aligned}$$

$$\begin{aligned} \therefore \bar{i} &= \frac{pV_0/r}{(p + 1/rk)(p + 1/RC)} \\ &= \frac{V_0/r}{1/rk - 1/RC} \left(\frac{1/rk}{p + 1/rk} - \frac{1/RC}{p + 1/RC} \right) \end{aligned}$$

$$\begin{aligned} \text{Now the Laplace transform of } \epsilon^{-\alpha t} &= \int_0^{\infty} \epsilon^{-t(\alpha + p)} dt \\ &= \frac{1}{\alpha + p} \end{aligned}$$

\therefore the current corresponding to \bar{i} is

$$i = \frac{V_0/r}{1/rk - 1/RC} \left(\frac{1}{rk} \epsilon^{-t/rk} - \frac{1}{RC} \epsilon^{-t/RC} \right)$$

$$\begin{aligned} \text{Now } V_k &= \int \frac{idt}{k} \\ &= \frac{V_0/rk}{1/rk - 1/RC} \left(\epsilon^{-t/RC} - \epsilon^{-t/rk} \right) \\ &= \frac{V_0}{1 - rk/RC} \left(\epsilon^{-t/RC} - \epsilon^{-t/rk} \right) \end{aligned}$$

The capacitor k continues to charge until $V_k = V$.

It is required to find this value.

APPENDIX

We have then

$$V_o \epsilon^{-t/RC} = \frac{V_o}{1 - rk/RC} \left(\epsilon^{-t/RC} - \epsilon^{-t/rk} \right)$$

whence

$$\epsilon^{-t/RC} = RC/rk \epsilon^{-t/rk}$$

hence

$$-t = \frac{\log_{\epsilon} RC/rk}{1/RC - 1/rk}$$

If rk/RC is written as ρ

$$\begin{aligned} \therefore V_k &= \frac{V_o}{1 - \rho} \left(\epsilon^{\frac{\log_{\epsilon} 1/\rho}{1 - 1/\rho}} - \epsilon^{\frac{\log_{\epsilon} 1/\rho}{\rho - 1}} \right) \\ &= \frac{V_o}{\rho - 1} \left(\epsilon^{\frac{\log_{\epsilon} \rho}{1 - \rho}} - \epsilon^{\frac{\rho \log_{\epsilon} \rho}{1 - \rho}} \right) \\ &= \frac{V_o}{\rho - 1} \left(\rho^{1/(1 - \rho)} - \rho^{\rho/(1 - \rho)} \right) \\ \therefore \frac{V_k}{V_o} &= \rho^{\rho/(1 - \rho)} \end{aligned}$$

For V_k to correspond to 5 on the P.P.M. scale when V_o corresponds to 7,

$$\frac{V_k}{V_o} = \frac{1}{2.51}$$

Whence it can be shown that

$$\rho = 0.84$$

rk is given as 2.5×10^{-3} seconds

$$\therefore RC = 3 \times 10^{-3} \text{ seconds}$$

$5\mu F$ and 600Ω give this value of time constant.

